NUMERICAL MODELLING OF WAVE CURRENT INTERACTION IN AN ESTUARY

by

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ABSTRACT

Numerical models to simulate the interaction of waves, currents and sedimenttransport are becoming standard engineering tool for design of coastal works. The paper presents a holistic model for coupled treatment of all these processes. A numerical model study of the transformation of waves propagating towards the Jade/Weser estuary in the German Bight is presented.

1. INTRODUCTION

Exact knowledge of the hydrodynamic and morphodynamic conditions is necessary for the design of coastal protection buildings as well as for the construction and the maintenance of harbors in the coastal area and in the mouths of estuaries. An estuary is a domain with very structured relief, where ocean waves may meet strong tidal currents and river flows. These currents may induce substantial changes in wave conditions and have significant influence on the distribution of wave energy. On the other hand, currents can be generated by water waves which have large gradients in the wave energy, such as may exist in the surf zone. Both hydrodynamic processes can act as sources for sediment transport processes. The sufficient description of the interaction between waves and currents build the basis for a better modelling of morphodynamic processes.

The theoretical framework to describe the wave-current interaction in form of radiation stresses is well understood due to the work by several pioneers (LONGUET-HIGGENS 1960 and WHITHAM 1962). The concept of the radiation stress is proven for the implementation in numerical simulation models as suitably. Particularly in coastal areas wave-induced currents can be well described by radiation stresses. The consideration of such wave-induced currents and the effects of waves in form of wave breaking on the sediment transport constitute a challenge for numerical simulation models in coastal engineering.

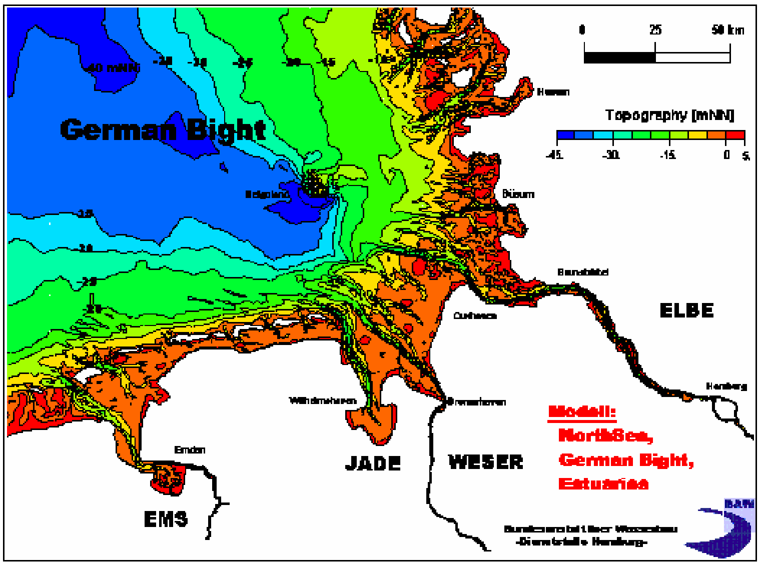

Figure 1 Estuaries in the German Bight

The German North Sea coast spans several estuary systems. For the design of new harbors (e.g. the JadeWeserPort) or the optimization of maintenance for existing harbors (e.g. Bremerhaven) the

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knowledge of the tidal and wave conditions as well as the morphological relocations to be expected constitute central questions in coastal engineering. In the paper a holistic model approach and its application to the Jade/Weser estuary are presented for the numerical simulation, which incorporates the interaction between currents, waves and morphology.

2. MODEL SET UP

In wave-current interacted flows, radiation stresses affect both wave and current fields. Wave energy may increase or decrease due to radiation stresses when tidal currents or river flows have significant spatial gradients. On the other hand, currents can be generated by water waves which have large gradients in radiation stresses, such as may exist in the surf zone. The description of morphodynamical events assumes an optimal approximation of the wave-current interaction. Variable depth conditions have an effect on the wave and current processes in turn. The coupled description of all processes is a prerequisite for realistic estimation of natural events.

Before the actual numeric model is presented, we must dealt with the meteorological conditions. The wind represents the driving forces of waves and currents. It is generally accepted that there is no backward influence from the wave-current field to the wind field. Thus the wind and pressure field may be modeled separately from all other water related state variables. Knowledge about the wind field is important for determining the wave properties which have to be prescribed at the boundaries of the areas under investigation. Inside the modeling areas the distribution of the wind- and pressure fields may be considered as external forces driving the currents and forming the waves.

The numerical model is based on a uniform mathematical formulation for currents, waves and sediment transport. All physical processes are represented in a single system of hyperbolic/parabolic differential equations which implies direct coupling of the different model components.

$$
\begin{aligned}
\frac{\partial K_i}{\partial t} &= -\frac{\partial \sigma_a}{\partial x_i} + C_g \frac{K_j}{k}\left(\frac{\partial K_j}{\partial x_i} - \frac{\partial K_i}{\partial x_j}\right) \\
\frac{\partial \sigma}{\partial t} &= -\left(U_i + C_{g_i}\right)\frac{\partial \sigma}{\partial x_i} - k_{x_i}\frac{\partial u_i}{\partial t} + f \cdot \frac{\partial z_B}{\partial t} \\
\frac{\partial a}{\partial t} &= -\frac{1}{2a}\frac{\partial}{\partial x_i}\left(U_i + C_{E_i}\right)a^2 - \frac{S_{ij}}{\rho g a}\frac{\partial U_i}{\partial x_i} + \frac{U_i\left(T_i - T_i^B\right)}{\rho g a} + \frac{\epsilon_B}{\rho g a} \\
\frac{\partial U_i}{\partial t} &= -U_j \frac{\partial U_i}{\partial x_j} - g\frac{\partial \bar{\eta}}{\partial x_i} - \frac{1}{\rho d}\frac{\partial S_{ij}}{\partial x_i} + \frac{1}{\rho d}\left(T_i - T_i^B\right) + c_w W_i |W| \\
\frac{\partial \bar{\eta}}{\partial t} &= -\frac{\partial U_j d}{\partial x_j} \\
\frac{\partial C}{\partial t} &= -U_i \frac{\partial C}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\tau_i \frac{\partial C}{\partial x_i}\right) + S \\
q_i &= \int_{-h}^{\eta} U_i C\, dz + q_{b_i} \\
\frac{\partial z_B}{\partial t} &= -\frac{1}{1-\kappa}\frac{\partial q_i}{\partial x_i}
\end{aligned}
\qquad (1)
$$

Here an orthogonal coordinate system is used with positive depth z_B. The components of the model system are described in the following sections.

2.1 HYPERBOLIC WAVE MODULE

The first four equations describe the evolution of wind waves, using the wave number vector $\vec{K}$, the angular frequency *σ* and the wave amplitude *a*. The propagation speed C, the group velocity C_g and the wave energy transport velocity C_E are obtained using linear wave theory. The wave model accounts for depth refraction as well as for current refraction. The input of turbulent energy in the water body due to wave breaking is accounted for by different breaking criteria. The breaking process of waves is represented by energy dissipation ϵ_B proportional to the difference between the actual and the stable wave energy flux (Dailly, Dean, Dalrymple 1985). To describe the interaction between currents the concept of radiation stresses S_{ij} are used. The tensor of radiation stresses is depending on wave parameters:

$$S_{ij} = E\left(\frac{C_g}{C}\frac{K_i K_j}{k^2} + \frac{1}{2}\left[\frac{2C_g}{C} - 1\right]\delta_{ij}\right)$$

(2)

where $\vec{K} = \left(K_x, K_y\right)$ is the wave number vector, k is the wave number and E is the wave energy. The energy dissipation by bottom friction is calculated as a period-averaged energy dissipation.

2.2 SHALLOW WATER FLOW MODULE

For engineering applications the current field is commonly modeled in terms of vertically integrated fluxes or velocities. The second block of equations represents the corresponding shallow water equations with the vertically integrated velocity vector $\vec{U}$ and the mean water elevation η. The total water depth is $d = z_B + \eta$ with the bed elevation z_B below the reference level. The current module accounts for the effects of wind shear stress at the surface with the wind drag coefficient c_W, the bottom friction T^B, a turbulence model by Smagorinsky, which takes into account the influence of turbulence energy input by wave breaking, the Coriolis forces, and the wave induced forces (radiation stresses).

2.3 MORPHODYNAMIC MODULE

Currents and waves act on the sea bed, picking up and releasing material. This material is transported by bed-load and in the water body as a concentration C of material. The last equation is the bottom-evolution equation which is used in order to calculate the morphodynamic changes. The parametric term $\left(1 - \kappa\right)$ represent the porosity of the bed material. Bed-load transport can be calculated by different formulations. In the simulations presented here, the formula of van Rijn (VAN RIJN, 1984) is used. The suspended sediment transport is solved by a depth integrated transport equation. For the source and sink term *S*, different formulations are possible. The maximum suspended sediment concentration is calculated by Rossinsy and Debolsky (ROSSINSY, 1980) in the simulation presented here. An extension of this single transport equation into a set of equations each representing a fraction of the grain size distribution curve and/or suspended and bed load transport would be straight forward.

The changing of the bed elevation couples back to the wave and current module due to the changing water depth.

2.4 NUMERICAL MODELLING

The derived equations are fully coupled. They are of hyperbolic/parabolic type. This allows the application of the same numerical algorithm to all equations.

The system of nine time-dependent partial differential equations is solved with stabilized finite elements (MILBRADT, 2002). Triangular grids are used with linear interpolation functions in space for the state variables. Higher order terms are removed by partial integration. The stabilization parameter corresponds to the largest absolute eigenvalues of the transport matrices in the above system of partial differential equations. The time integration is performed in the usual step by step manner.

The application of triangular elements guarantees good reproduction of the changing depth structure which is important for tidal and wave modeling. The mesh size can be determined by the bathymetrical complexity because the present model is based on the period-averaged form of the elementary equations. In regions where the wave field has complex structure, the mesh size is limited by the mean wave length.

As all differential equations are of propagation type, the same numerical scheme is being applied to all equations. Thus a short and clearly structured coding is possible. As the solution procedure has been reduced to an explicit scheme, the computational expense is linearly proportional to the number of equations and terms involved as well as to the number of elements or nodes of the discrete system.

2.5 INPUT DATA

Numerical simulation models are commonly driven by boundary conditions. These have to be specified in agreement with the simplifications made within the models.

Wind data are mostly available for few locations only. Interpolation to the model areas is difficult in case of modification of the wind field near structured and high coastlines. Alternatively data's from meteorological models can be used. The modification of this data due to the different roughness of land and water in the domain of investigation is also necessary (PLÜß, 2001).

Wave data have to be obtained from field measurements in which quite different processes my be superimposed. Thus the data analysis is tricky and time consuming. The accuracy remains limited. Determination of wave direction for instance is realistic only within an error of about 10 degrees.

Knowledge about currents at boundaries require field campaigns. As these are very expensive quite often water level measurements are taken and accepted. With the permanently increasing quality of hindcast models the results of such models can be used for the water level as well as the current velocity to model control.

Morphodynamic processes are very difficult to monitor. It is nearly impossible to do echo sounding for large areas continuously. Generally data have to be taken which are available anyway.

Beside the scarcity of data there are two further problems. The first concerns the fact that only few synoptic data are available and the second aspect concerns the selection of input data with respect to the intended purpose of a study.

3. MODELAPPLICATION

The Jade/Weser estuary is located in the center of the German Bight and is separated in the Jade bay without fresh water discharge and the long winded riverlike Weser. The estuary system consists of major navigation channels, extensive inter-tidal banks and complex coastlines. With the planning of the JadeWeserPort and the expansion of the container terminal Bremerhaven consolidated knowledge about the hydro- and morphodynamics in the Jade/Weser estuary are required.

Within the scope of the preplanning of the JadeWeserPort different investigations have been done by several institutions to quite different questions. Investigations to the tidal dynamic in particular to the storm character were carried out by the BAW (PLÜß 2001). The wave climate in the Jade/Weser estuary is investigated by the Franzius-Institute (MAI 2000) were carried out under disregard of the tide influence. Investigations to the interaction of waves and tidal currents are open for the Jade/Weser estuary yet.

In a numerical study the holistic model is used for an analysis of the wave propagation in the Jade/Weser estuary with considerations of the prevailing tidal conditions. Special attention should lie on possible effects for future morphodynamical modelling.

3.1 MODEL AREA

Basis of the numerical simulation model of the Jade/Weser estuary was a digital terrain model provided by BAW. The computational grid was automatically generated with the preprocessor Janet (SMILECONSULT 2002). The quality of the depth reproduction of the computational model grid can be controlled by specifying acceptable relative and absolute depth differences. During the automatic mesh generation process, further optimization of the geometrical and topological quality of the model grid is performed.

The model mesh for the Jade/Weser estuary consists of 76158 nodes and 148205 elements.

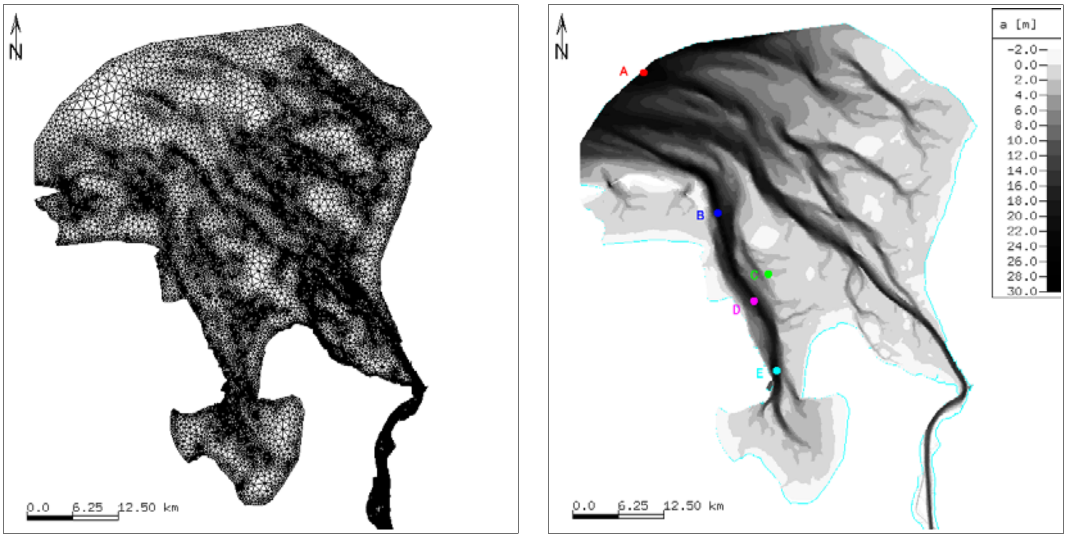

Figure 2: finite element mesh and depth distribution of the Jade/Weser model

The boundary conditions necessary for the model control come from different sources. Mean water level and mean current velocities at the open sea boundary come from the German Bight model of BAW and control the current module. Also the discharge of the Weser river come from investigations at BAW. To control the wave module sea-sided boundary values are necessary in form of mean wave amplitude, period and direction. Moreover model data of the BSH as well as from measurements were available. The wind is taken into consideration both in the current module and in the wave module as a driving force and was provided by DWD.

3.2 CASE STUDY

For the presented numerical model study a hypothetical event is considered. Starting with a mean tidal event, a quasi stationary wind and wave field are used to couple with the tidal and morphodynamic investigation. In a second step the use of a natural measured event is planed. The water surface elevation of the mean tidal event at the boundaries and the interval of analysis between flood and ebb are shown in Figure 3.

For the numerical study a north north west wind situation is assumed with constant wind speed of 7 m/s. To control the wave module boundary conditions for waves with a height of 1m, a period of 4 s and a mean direction same as the wind direction are used.

The time variations of wave height at several positions (see Figure 2) computed by the holistic model are shown in Figure 3, which demonstrates the significance of tidal variation effects on wave transformation. The result for position C, which is located near tidal flats, show that the variation of water depth has a profound influence on wave transformation. The time variation of wave conditions in deep water and tidal channels is also found to be significant (see positions B and D), primarily due to changes in tidal current pattern.

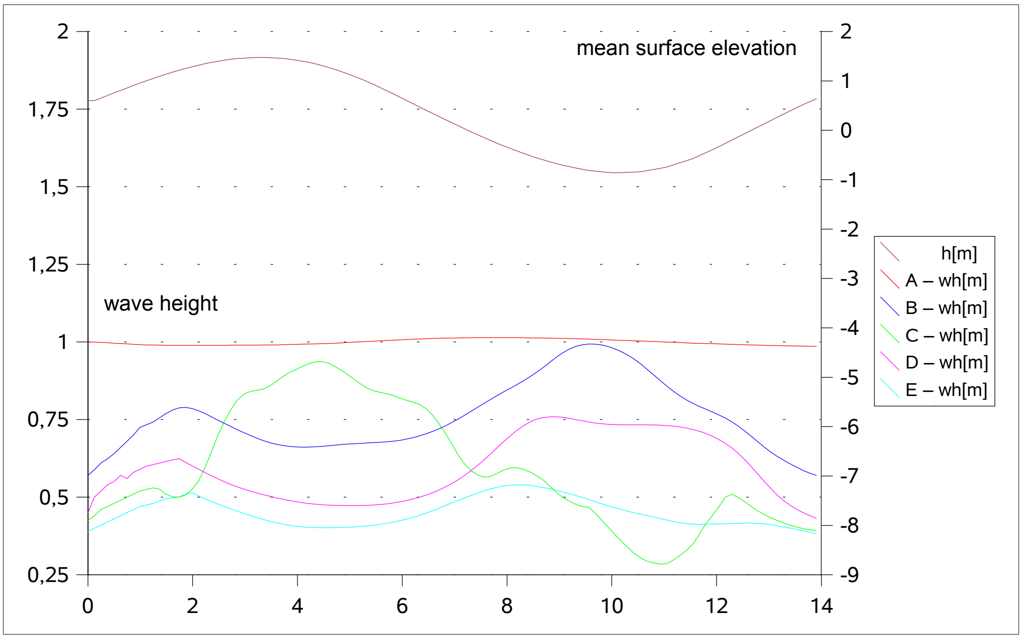

Figure 3: Time variation plots of mean surface elevation(above) and wave height (bottom) at several location

The Figures 4 and 5 show the study domain (contour plot of bathymetry) and characteristic wave parameters during maximum ebb and flood tidal flow. Waves are carried by the tidal currents. During ebb flow, wave energy is blocked by the currents. During flood tide, the waves advance considerably farther into the estuary. This phenomenon is further enhanced during storm surges, which strongly raise water levels.

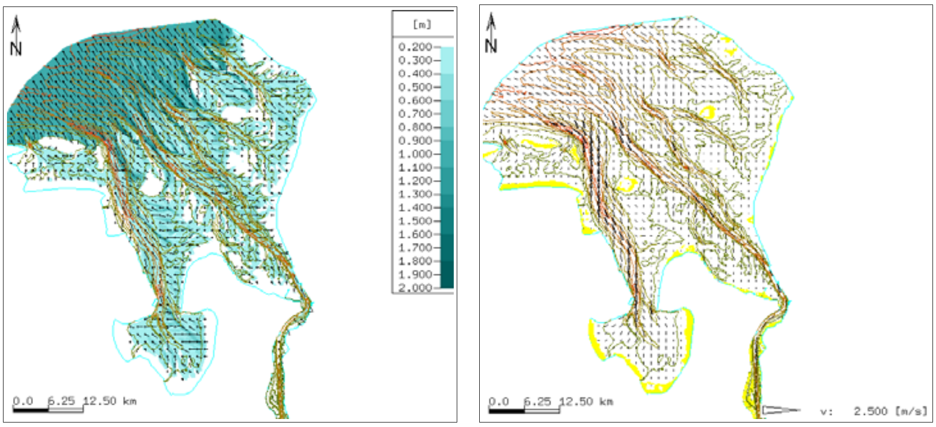

Figure 4: Wave distribution and current patterns at maximum ebb flow

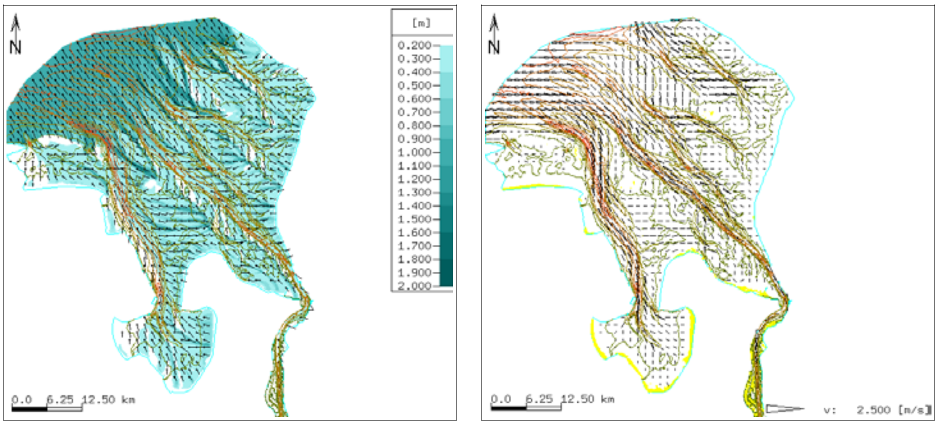

Figure 5: Wave distribution and current pattern at maximum flood flow

The energy loss of the waves, basically by the breaking of the waves in shallow areas, produces wave induced currents (often close to coast). These wave induced currents can modify the tidal currents and can lead with sediments to different transport patterns.

In the Figure 6 the transport capacity of the sediments is shown for the flood situation. If the effect of the wave field is neglected in the numerical model, the sediment transport capacities concentrate basically upon the tide channels of the estuary. If a coupled simulation is carried out, taking into account the wave influence we can observe essentially larger areas affected by morphological activities. An essential reason for this is the wave breaking on the intertidal flats.

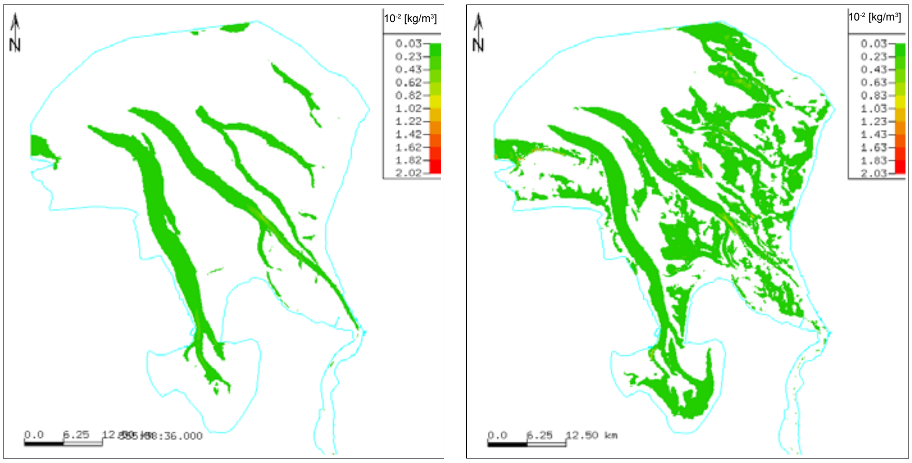

Figure 6: distribution of sediment-transport capacities by flood, without and with wave interaction

The evaluation of resulting depth changes, which are calculated by long time simulations allow to expect that here, too, differences appear between calculations with wave influence and without. By the difference of the calculated relocation between the consideration of the tidal currents on the one hand and the coupled tidal and wave induced currents on the other hand, areas and dimensions can be defined, where the morphological effect of the waves is significant and must be continuously examined.

4. CONCLUSION

The simulations show that considerable wave conditions exist in an estuary during the whole tidal cycle, not only on the tidal flats but also in the deeper areas where variations in the currents are significant. Additional first morphodynamical simulations are presented with and without wave influence. It was shown that with interaction of waves and currents within the scope of numerical simulation of morphodynamical processes, essential temporal and spatial differences exist in the sediment transport capacities. Further investigations with natural scenarios are planned.

5. ACKNOWLEDGMENTS

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