Modelling wave propagation in large areas

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1 Introduction

The aim of coastal engineering is to estimate the effects of coastal protection structures. The erosion of coastal sections requires measures to regulate the sedimentbudget. Beach protection works and coastal structures are designed according to the local wave conditions.

During the planning phase it is necessary to have the appropriate instruments to estimate the effects of building measurements. There exist theoretical procedures as well as hydrological and numerical models. In the past the trend goes uniquely to the increase of the use of computers.

The commercial models - corresponding to their desired task - are based on different calculation assumptions and solution algorithms and therefore they are not of universal use.

For their analysis large areas have to be modelled in order to determine the wave characteristics of deep water waves from different origin and direction propagating into shallow water near shore regions. Numerical treatment of nonlinear waves for such areas of some ten kilometers in extension is beyond the capabilities of workstations. A compromise between numerical practicability and physical quality of the results may be based on linear wave theory. This was shown by field measurments and statistical analysis even for a near-shore groinfield [5].

The following presented numerical wave model is a part of the program system TICAD (Tidal Interactive Computation And Design) [12]. It was developed for large-scale areas. The range of application encloses deep and shallow water regions up to breaking zones.

2 Wave model: Theoretical background

The wave model is based on the numerical solution of analytical and empirical approximation functions as they are given for instance in Shore Protectional Manual [1].

On the basis of the linear wave theory of AIRY propagation and changes of a monochromatic wave are calculated by the wave front method. In the case of neglecting external forces (for example wind forces) and of breaking zones the method is based on the conservation of mean energy flux between two wave normals. The mean flux of energy is proportional to the product of the group velocity c_g and the square of the wave height H:

$$F = c_g * \frac{\rho * g * H^2}{8} \tag{1}$$

Under deep water conditions ($d \geq 0.5 * L$) the wave propagates with constant velocity c_0 (index 0 indicates deep water conditions). In this case the wave length L_0, periode T_0 and height H_0 keep their values.

The wave parameters, except the period T, will change if the orbital motion of the wave touches the bottom ($d \leq 0.5 * L$) or if the wave will meet an obstacle like a mole or an end of an island or if it enters into a region with currents. These influences are called shoaling, refraction, diffraction and current-refraction. They are taken into account by the wave model as well as breaking of the waves due to very low water depth and steepness of the wave. In addition the consideration of a windfield is possible. The influences of perculation and reflection are neglected.

The necessary information about the bathymetry of a coastal region are taken from the digital terrain model (DTM). The DTM allows the fitting at every arbitrary terrain bathymetry due to triangular latticenet with variable width of the meshs. On the basis of that latticenet the tidal induced current are calculated by a 2-dimensional hydrodynamic model based on FEM. The result of this tidal calculation (current, water level) serves as input for the wave model.

3 Calculation of the wave parameter

Wave length

The wave length is calculated by the implicitly equation:

$$L = \frac{g * T^2}{2 * \pi} * tanh\left(\frac{2 * \pi * d}{L}\right) \tag{2}$$

Shoaling

Entering more shallow water the wave begins to 'feel bottom', when the water depth is about one half of the wave length. The waves are hereafter slowed, shortened and steepened, as they travel into more shallow water. This process is called shoaling. The group velocity is calculated by

$$c_g = c * n = \frac{c}{2} * \left[1 + \frac{\frac{4*\pi*d}{L}}{sinh(\frac{4*\pi*d}{L})}\right] \tag{3}$$

with

$$c = \frac{L}{T}. \tag{4}$$

The shoaling coefficient k_s is calculated for an arbitrary terrain point by

$$k_s = \frac{H_2}{H_1} = \frac{c_{g1}}{c_{g2}} \tag{5}$$

$$= \sqrt{\frac{L_1 * T_2}{L_2 * T_1} * \frac{\left[1 + \frac{\frac{4*\pi*d_1}{L_1}}{sinh\left(\frac{4*\pi*d_1}{L_1}\right)}\right]}{\left[1 + \frac{\frac{4*\pi*d_2}{L_2}}{sinh\left(\frac{4*\pi*d_2}{L_2}\right)}\right]}}. \tag{6}$$

Depth-Refraction

If the wave front in shallow water meets a bottom contour at an angle, the direction of travel is changed. This process of refraction is due to the fact, that water waves propagate more slowly in shallow than deeper water, and therefore the front tends to get aligned with the contours. This phenomenon is comparable with the refraction of the light described by the $SHNELL$'s refraction law. The change of the wave height due to refraction is calculated using a formula from $WIEGEL$ [11]:

$$k_r = \frac{H_2}{H_1} = \sqrt{\frac{b_1}{b_2}} \tag{7}$$

with b =distance between the orthogonals.
Refraction appears always in superposition with shoaling.

Current-Refraction

Changes of phase velocity can also be caused by currents, resulting in current-refraction. In case of coincidence between current and wave propagation direction the wave length will be decreased and the wave height will be increased. Opposite directions cause reverse effects. The influence of the current is calculated using an approach of $JOHNSON$ [6]. In Fig.1 the geometrical relations are described for the general case. A wave propagates under the angle α from still to flow water region. The change of the wave parameters are calculable using the geometrical conditions at the discontinuation surface, assuming that the change of the velocity is a jump.

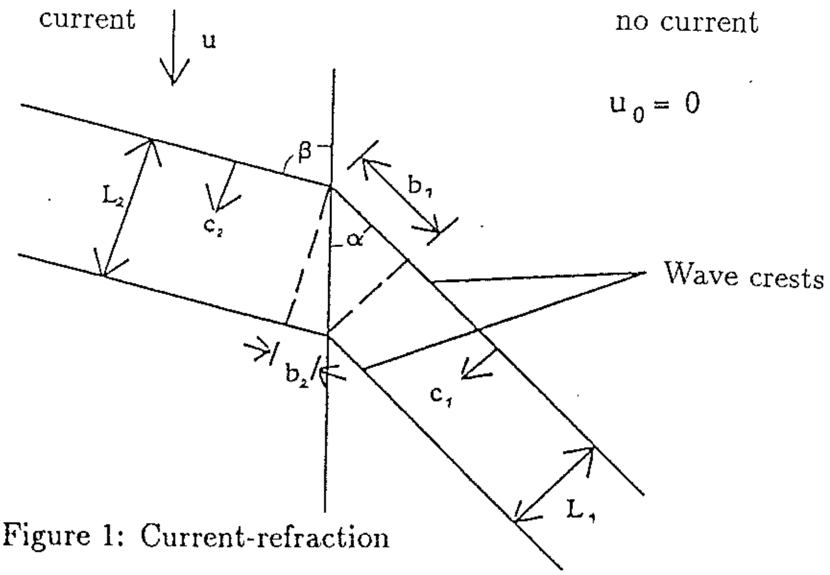

Figure 1: Current-refraction

Hence, the following relations are valid:
wave length

$$\frac{L_1}{sin(\alpha)} = \frac{L_2}{sin(\beta)} \tag{8}$$

propagation velocity

$$\frac{c_1}{sin(\alpha)} = u + \frac{c_2}{sin(\beta)} \tag{9}$$

With the conservation of enery flux and the above equations the change of the wave height is calculated:

$$k_{st} = \frac{H_2}{H_1} = \sqrt{\frac{c_{g_1} * b_1}{(c_{g_2} + u * sin(\beta)) * b_2}} \tag{10}$$

Diffraction

Diffraction is the propagation of a wave behind an obstacle as a mole or an end of an island. In analogy to the geometrical optic the change of wave height is calculated using the equation of *SOMMERFELD*.

Fig.2 shows the relations between the angles and equation (11) is the solution of the *SOMMERFELD*-equation.

$$F(r,\theta) = f(\sigma) * e^{-ikr*cos(\theta-\theta_0)} + f(\sigma') * e^{-ikr*cos(\theta-\theta_0)} \tag{11}$$

with

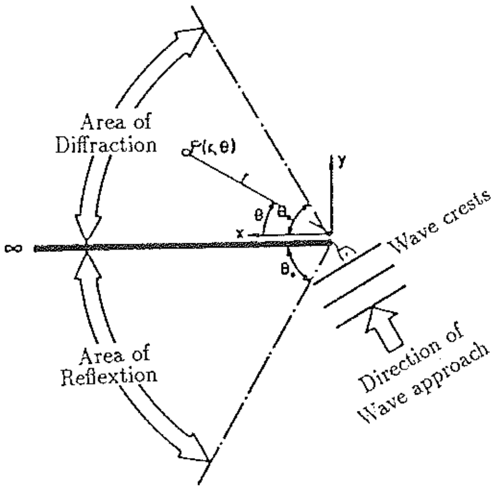

Figure 2: Diffraction

$$\sigma = 2 * \sqrt{\frac{k * r}{\pi}} * sin\left(\frac{\theta - \theta_0}{2}\right) \quad (12)$$

$$\sigma' = -2 * \sqrt{\frac{k * r}{\pi}} * sin\left(\frac{\theta - \theta_0}{2}\right) \quad (13)$$

$$f(\sigma) = \frac{1+i}{2} * \int_{-\infty}^{\sigma} e^{\frac{-i\pi t^2}{2}} dt \quad (14)$$

$$f(\sigma') = \frac{1+i}{2} * \int_{-\infty}^{\sigma'} e^{\frac{-i\pi t^2}{2}} dt \quad (15)$$

$$k = \frac{2 * \pi}{L} \quad (16)$$

$$i = \sqrt{-1} \quad (17)$$

The diffraction coefficient k' is the ratio of local to incoming wave:

$$k' = \frac{H(r,\theta)}{H_0} = \mid F(r,\theta) \mid \quad (18)$$

In the wave model the effect of diffraction with effect of shoaling, refaction and current-refaction are interacting, therefore is valid:

$$H_2 = k_s * k_r * k' * k_{st} * H_1. \quad (19)$$

Breaking of the waves

The breaking of the waves due to very large steepness can be checked using the generalised $MICHE$-criteria in consideration of shallow water conditions [1]:

$$\frac{H_B}{L} = 0.14 * tanh\left(\frac{2 * \pi * d}{L}\right) \quad (20)$$

The index B is considering the breaking conditions.

The assumption from $WEGGEL$ [10], which is derived from the analysis of a lot of labor experiments corresponds to the breaking due to very low water depth independence on the underwater beach slope:

$$\frac{H_B}{d} = b - \frac{a * H_B}{g * T^2} \tag{21}$$

with

$$a = 43.75 * \left(1 - e^{-19*m}\right) \tag{22}$$

$$b = \frac{1.56}{1 + e^{-19*m}} \tag{23}$$

$m =$ is the underwater beach slope

Both breaking criteria are checked by the wave model. The change of the wave height due to energy lost during breaking will be calculated using an approach from *HORIKAWA/KUO* [7] and *ANDERSON/FREDSOE* [4] . In the distance x behind the breaking line one gets the following wave height formula:

$$\frac{H}{H_B} = 0.35 + 0.65 * e^{\frac{-0.12*x}{H_B}} \tag{24}$$

Bottom friction

For waves advancing in still water it is usual to assume that the variation in height with distance may be represented, locally, by

$$H(x_2) = H(x_1) * e^{a_0*(x_2-x1)} \tag{25}$$

where a_0 is a wave attenuation coefficient. The coefficient a_0 is approximated by

$$a_1 = \frac{\left(\frac{2*\pi}{L}\right)^2}{\sqrt{\frac{\pi}{T*\nu}} * \left(\frac{2*\pi*d}{L} + sinh\left(\frac{2*\pi*d}{L}\right)\right)} \tag{26}$$

where ν is the kinematic viscosity and a_1 is the component of a_0 attributable to energy dissipation in the boundary layer at the bed [8]. The bottom friction has the status of lower importance for problems of coastal engineering.

Wind field effects

In most coastal locations in the world no wave records deriving from field measurements are available and both the time scale for the design and financial resources are available make the installation and operation of such devices unfeasible for at least one climatological year.

The dominant wave parameters are generally computed a priori and if possible supported by several measurements. Using the a priori calculation you have the possibility of deterministic and statistic methods. Using the deterministic method the parameters of a "decisive" wave, in generally from engineers point of view the important $H_{1/3}$ value and the accessory period are defined. The statistic methods are describing the totality of waves in a field of waves. The North Sea is a typical case of the "Jonswap-Spectral" based on measures. Both methods have not implemented any information about the effects of bathymetry and the shore for the wind direction. By the numerical modelling there are possibilities given, to calculate the development of wave height also in shallow water areas with complicated depth partitionation, by implementation of refraction, shoaling, diffraction and breaking. Also it is possible to get information about the influence of realistic wind events with differences in surface and time. The program calculates the change of the significant waveheight $H_{1/3}$ and wave priod in an wind field caused by wind appropiated by [1]:

$$\frac{gH}{U_A^2} = 0.283 * tanh\left[0.53 * \left(\frac{gh}{U_A^2}\right)^{\frac{3}{4}}\right] * tanh\left\{\frac{5.65 * 10^{-3} * \left(\frac{gF}{U_A^2}\right)^{\frac{1}{2}}}{tanh\left[0.53 * \left(\frac{gh}{U_A^2}\right)^{\frac{3}{4}}\right]}\right\} \quad (27)$$

$$\frac{gT}{U_A} = 7.54 * tanh\left[0.833 * \left(\frac{gh}{U_A^2}\right)^{\frac{3}{8}}\right] * tanh\left\{\frac{3.79 * 10^{-2} * \left(\frac{gF}{U_A^2}\right)^{\frac{1}{3}}}{tanh\left[0.833 * \left(\frac{gh}{U_A^2}\right)^{\frac{3}{8}}\right]}\right\} \quad (28)$$

with U_A = wind velocity
F = wind fetchlength.

The Jonswap-equations can be calculated alternatively.

In the program, the above equation have been modified. Now the differences of wave height along the running direction are calculated. On this way the influence of bathymetry and the shore course on the development of sea violence can be considered by observing the waves step by step. In the actual program release the uniform wind conditions for the target area can be calculated.

Wave induced streaming

The streaming in nature are superpositon of tide and wave induced streaming which are interacting.

Now one can calculate the induced forces on the background of socalled radiation stresses by $LONGUENT - HIGGINS$ and $STEWART$ [9].

It is convenient to investigate how a progressive wave contributes, through the induced horizontal momentum and pressure components, to the dynamic equilibrium of a water column and to define and formulate the radiation stress magnitudes. The magnitudes will be used in the circulation models.

Independently of the first order wave theory postulation that waves transport no mass in the direction of their propagation due to the periodicity and symmetry of the velocity u magnitude, there is a surplus of momentum flux showing that gradients of induced mean momentum.

Now one can calculate the socalled radiation stresses

$$S_{xx} = \left(n * cos^2(\theta) + n - \frac{1}{2}\right) * E \quad (29)$$

$$S_{xy} = (n * sin(\theta)cos(\theta)) * E \quad (30)$$

$$S_{yy} = \left(n * sin^2(\theta) + n - \frac{1}{2}\right) * E \quad (31)$$

$$(32)$$

where, $E = \frac{1}{8}\rho g h^2$, the wave enery, $n =$ the ratio of group velocity to wave celerity and θ is the angle between the direction of wave propagation and the positive x-axis.

The driving forces are found from the gradients of the wave action, leading to

$$F_x = \frac{1}{\rho * h} * \left(\frac{\delta S_{xx}}{\delta x} + \frac{\delta S_{xy}}{\delta y}\right) \quad (33)$$

$$F_y = \frac{1}{\rho * h} * \left(\frac{\delta S_{xy}}{\delta y} + \frac{\delta S_{yy}}{\delta y}\right) \quad (34)$$

$$(35)$$

With this forces one can calculate the induced velocity by a streaming model.

4 Test cases and applications

The wave program calculates the change of the wave parameters H, L, T, the direction and the possible breaking of a monochromatic wave for the cutting points between wave crests and orthogonals. In this case the quality of this approach is dependent on choosing the step of calculation in dependent on the task. The wave parameters are constant between two orthogonals. The results of the calculations can be written in the following ways:
1. Representation of the wave crests with and without orthogonals in 7 colours indicating the wave height reaches. The boundaries of this reaches can be choosen freely.
2. Output of the wave parameters including the streaming induced forces at determinate points, at the so called pegel points.
3. The same information can output at the points of the digital terrain model.

The calculation of the wavepropagation by the linear wave theory gives good results for a lot of practical cases. In the following for some examples the precise restitution of the model is demonstrated.

As an example the test region is modelised in the same way as published by *DE VRIEND* in [2] and [3]. This is a system with a curved coastline. The distribution of the depth is represented in Fig. 3. For this region the waves simulation is calculated.

The wave distribution is given in Fig. 4, the dotted region shows the breaking of waves. Using in Fig. 6 represented components of forces the wave induced streaming from Fig. 7 is calculated as an example. The correctness of the calculation be examaind in the case of the above presented test region. Of course it is necessary a more complete verification by other natural data.

A large scale area application for this model is the coastal protection investigations in the coastel region near the island of Sylt. Sylt is located in the south-east part of the North Sea near the border between Germany and Danmark. In Fig. 7 the distribution of the depth is represented. Besides the effects according to the bathemetry one can good seen in Fig. 8 and Fig. 9 the effects of tidal streaming at the ends of the island.

The model reproduces well-known results for finite-amplitude waves.

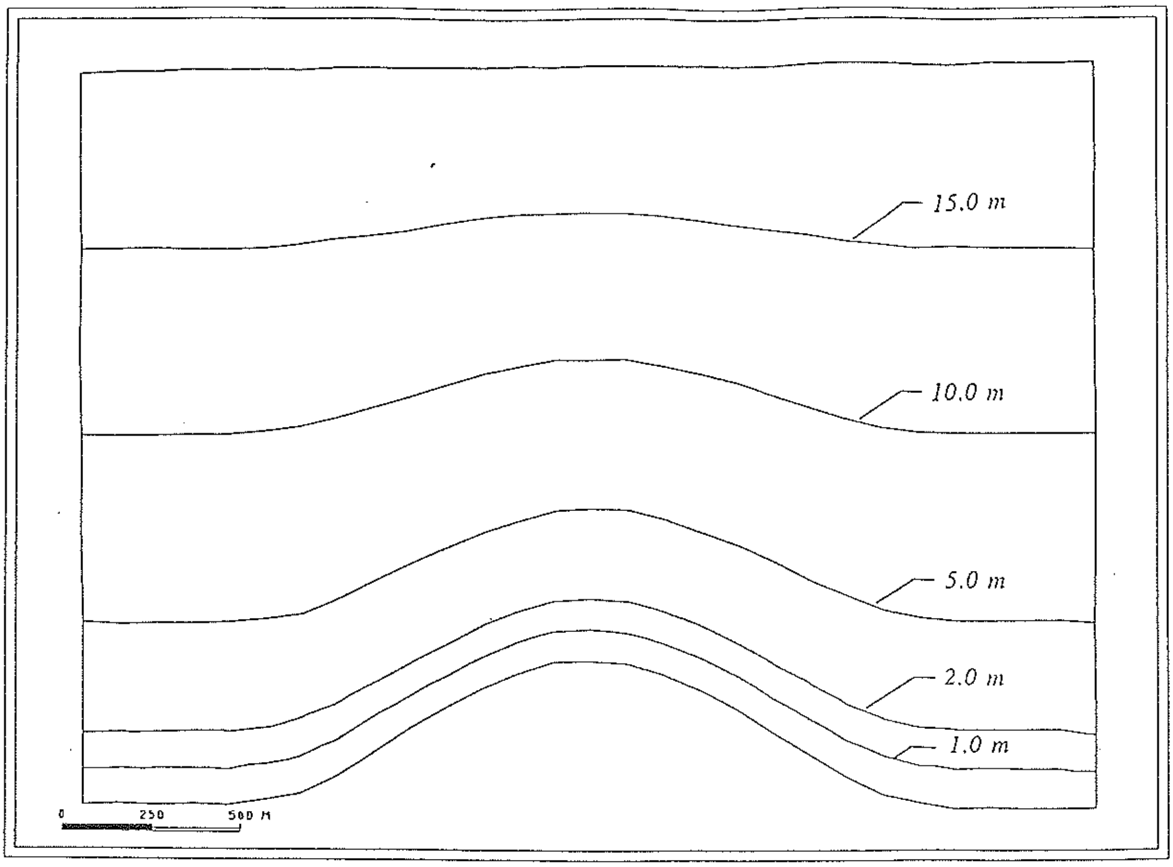

Figure 3: Bathematry of testregion

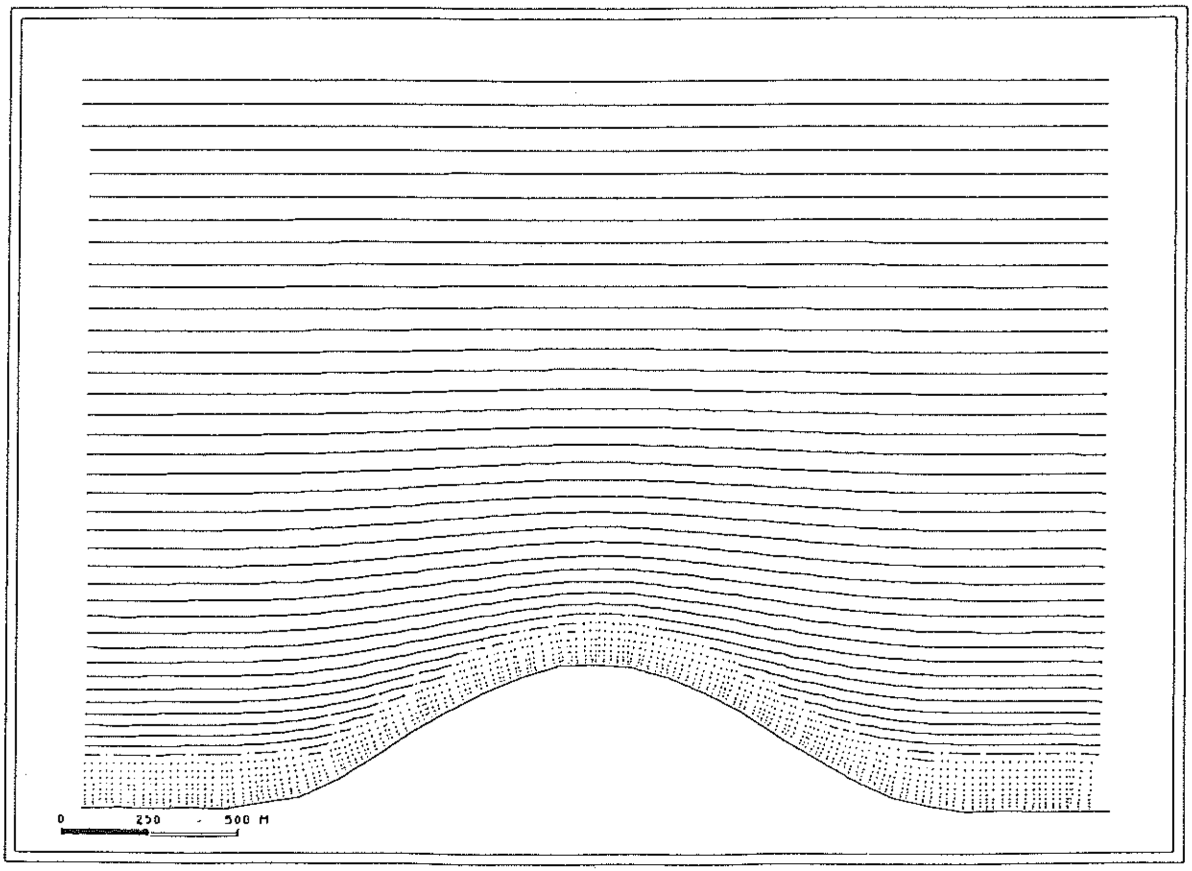

Figure 4: Wave distribution

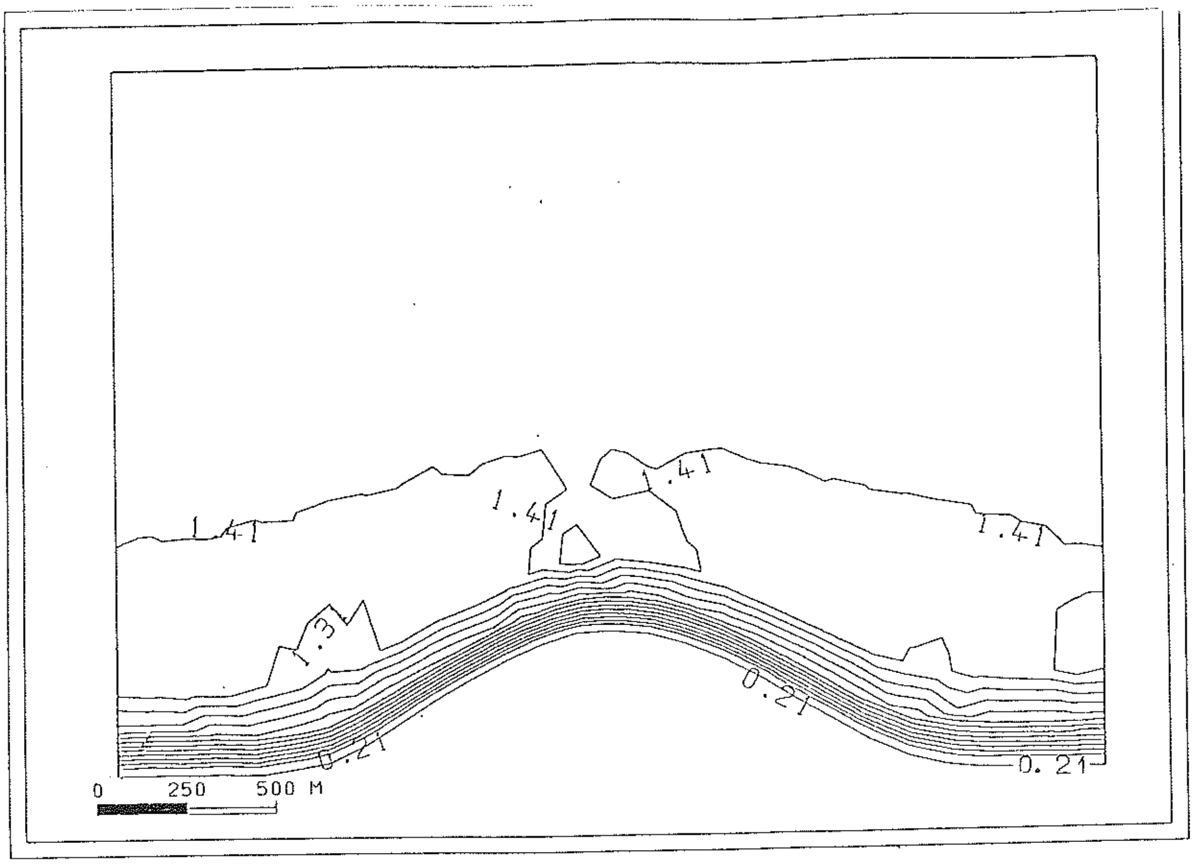

Figure 5: Wave height distribution

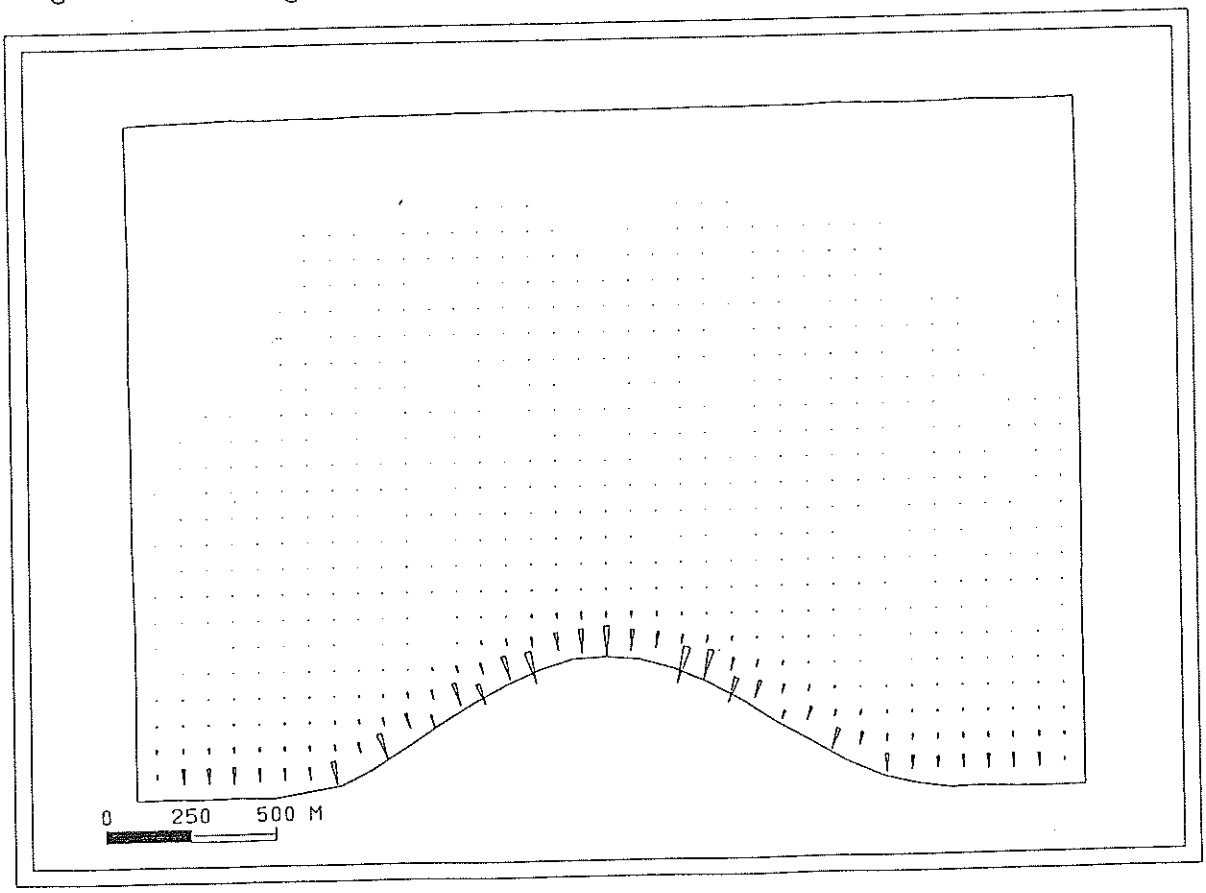

Figure 6: Current induced forces

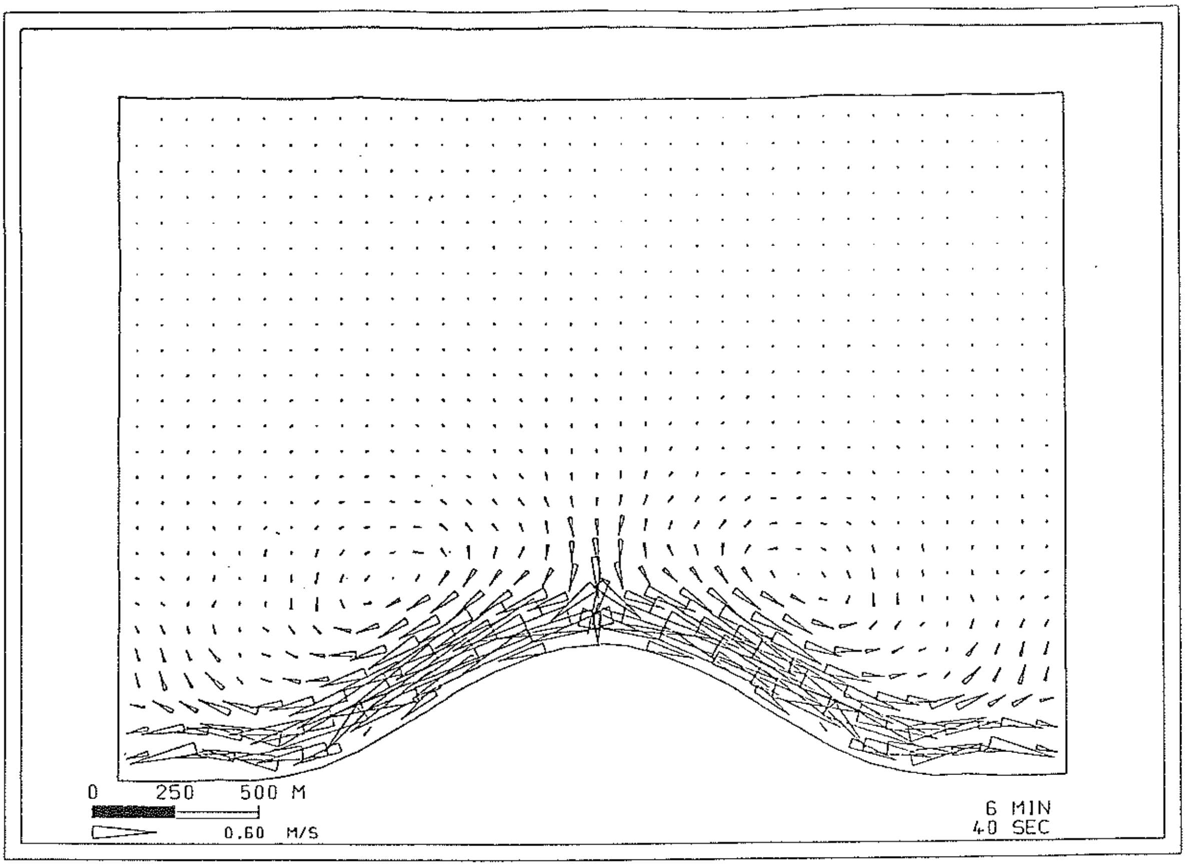

Figure 7: Wave induced currents

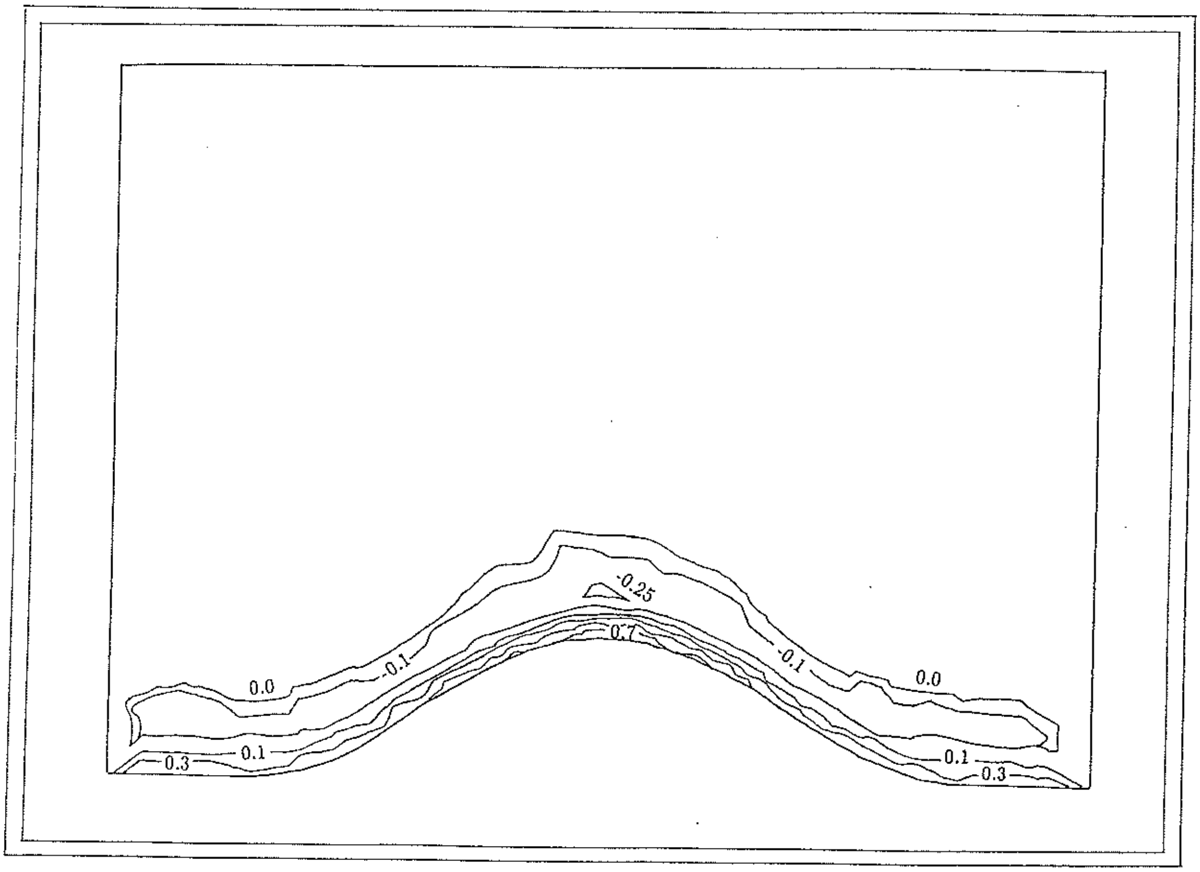

Figure 8: Mean Waterlevel because of waveinfluence

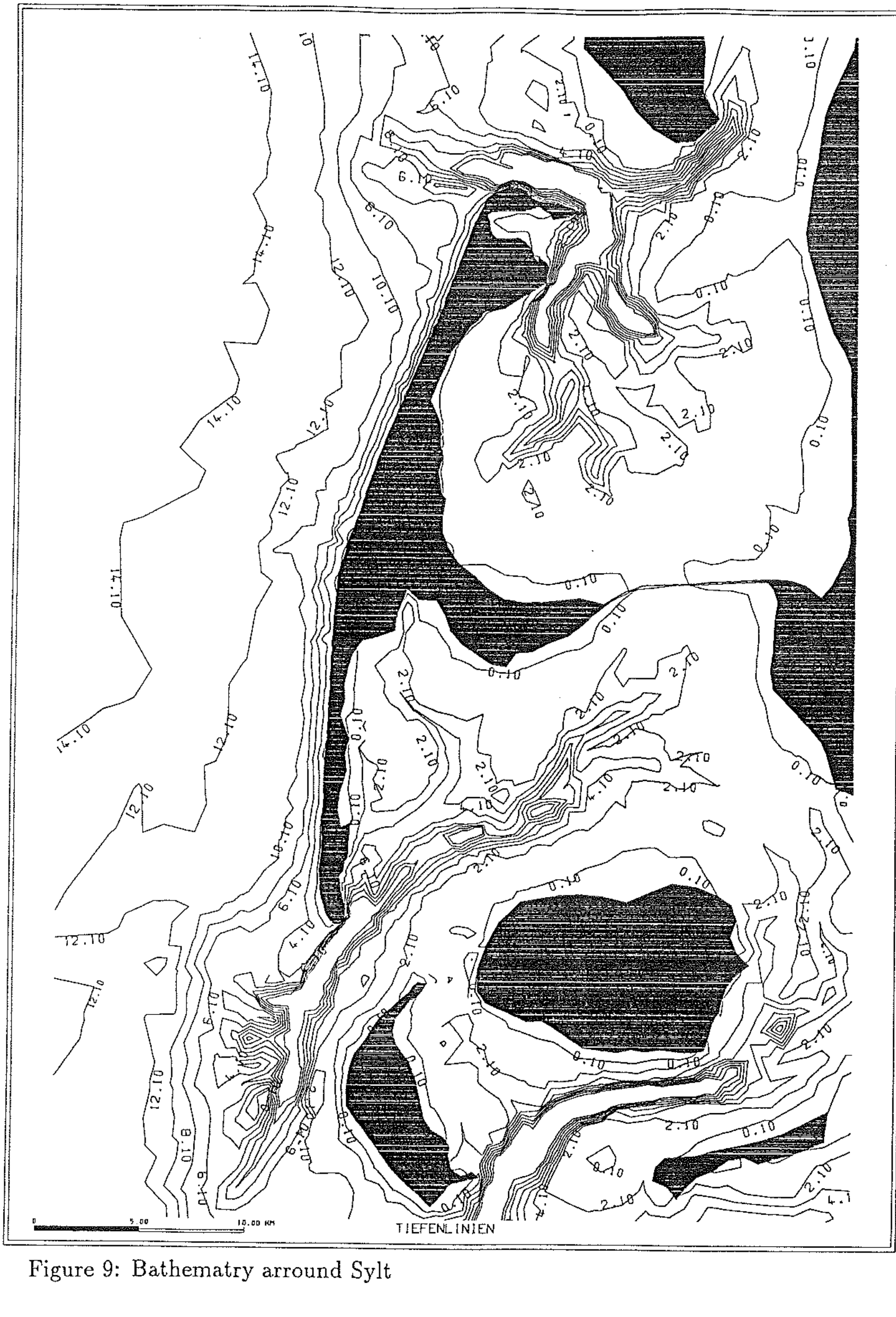

Figure 9: Bathematry arround Sylt

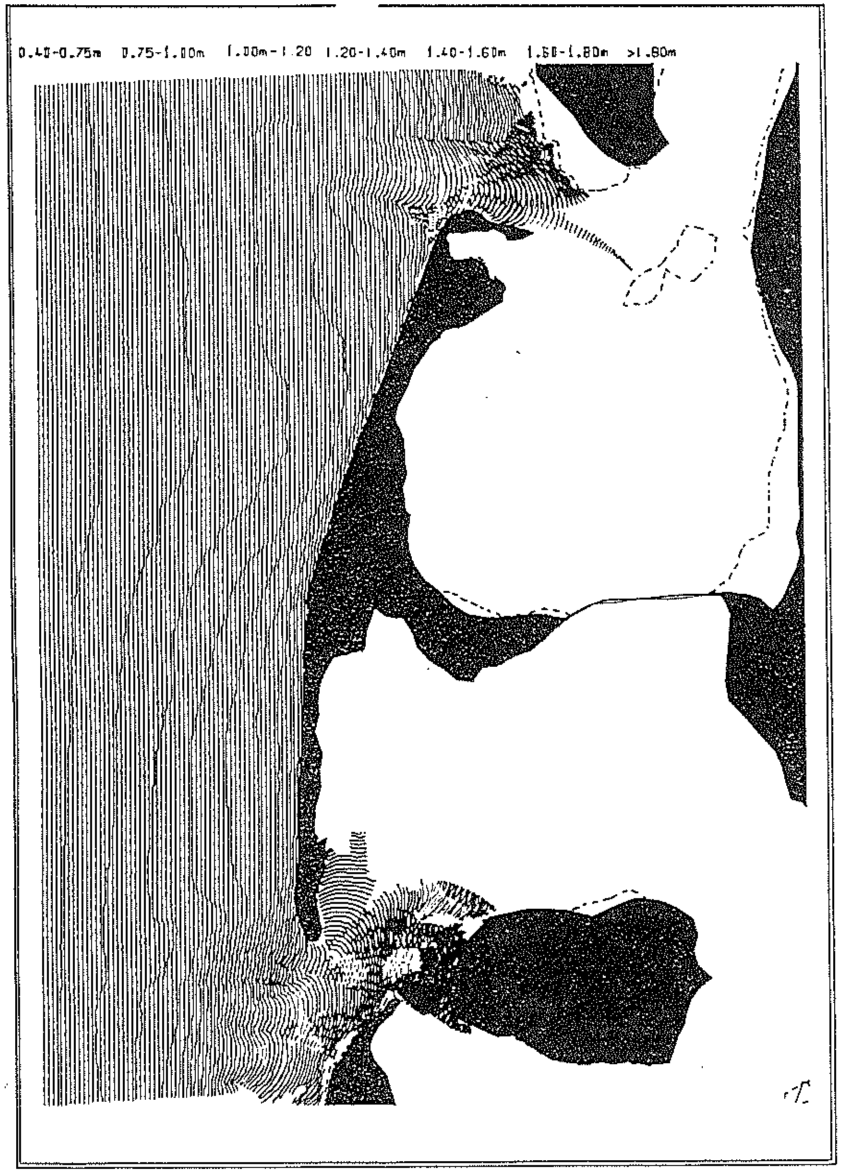

Figure 10: Wave distribution during ebb phase

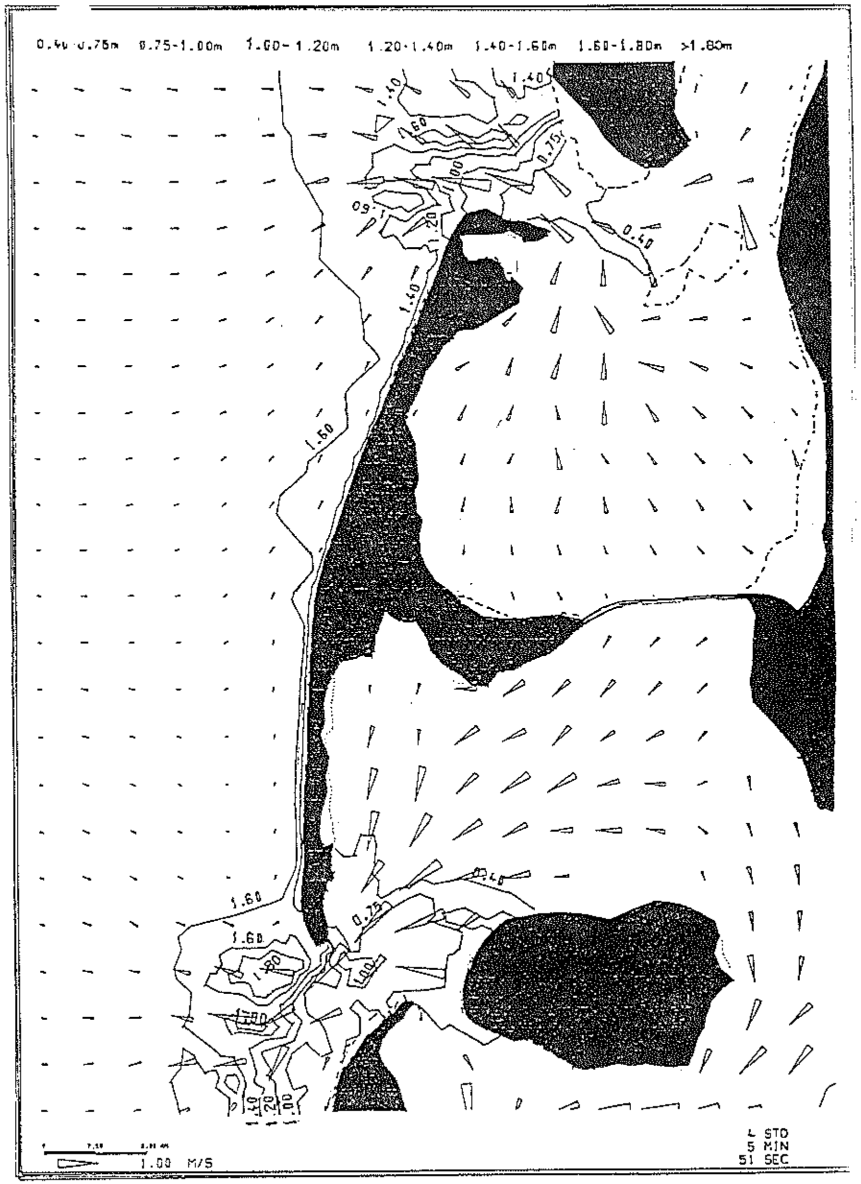

Figure 11: Wave height distribution and current velocity during ebb phase

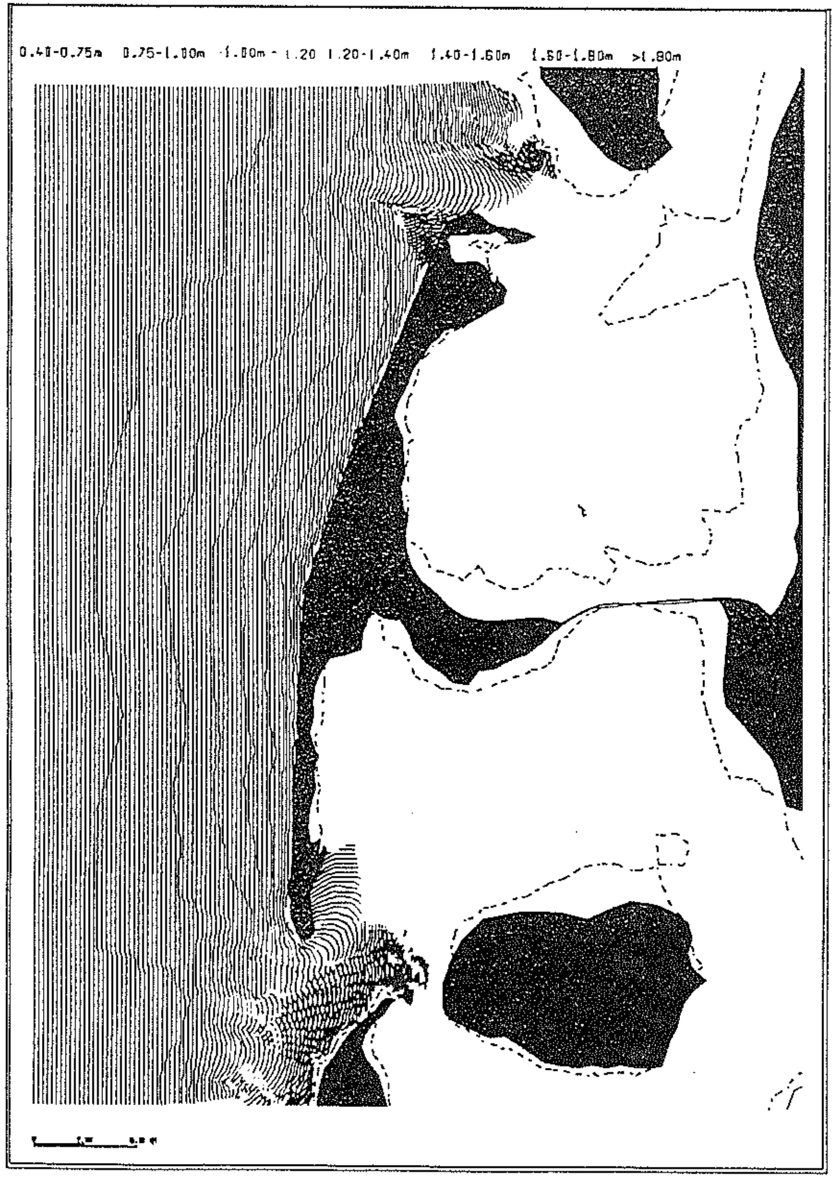

Figure 12: Wave distribution during flood phase

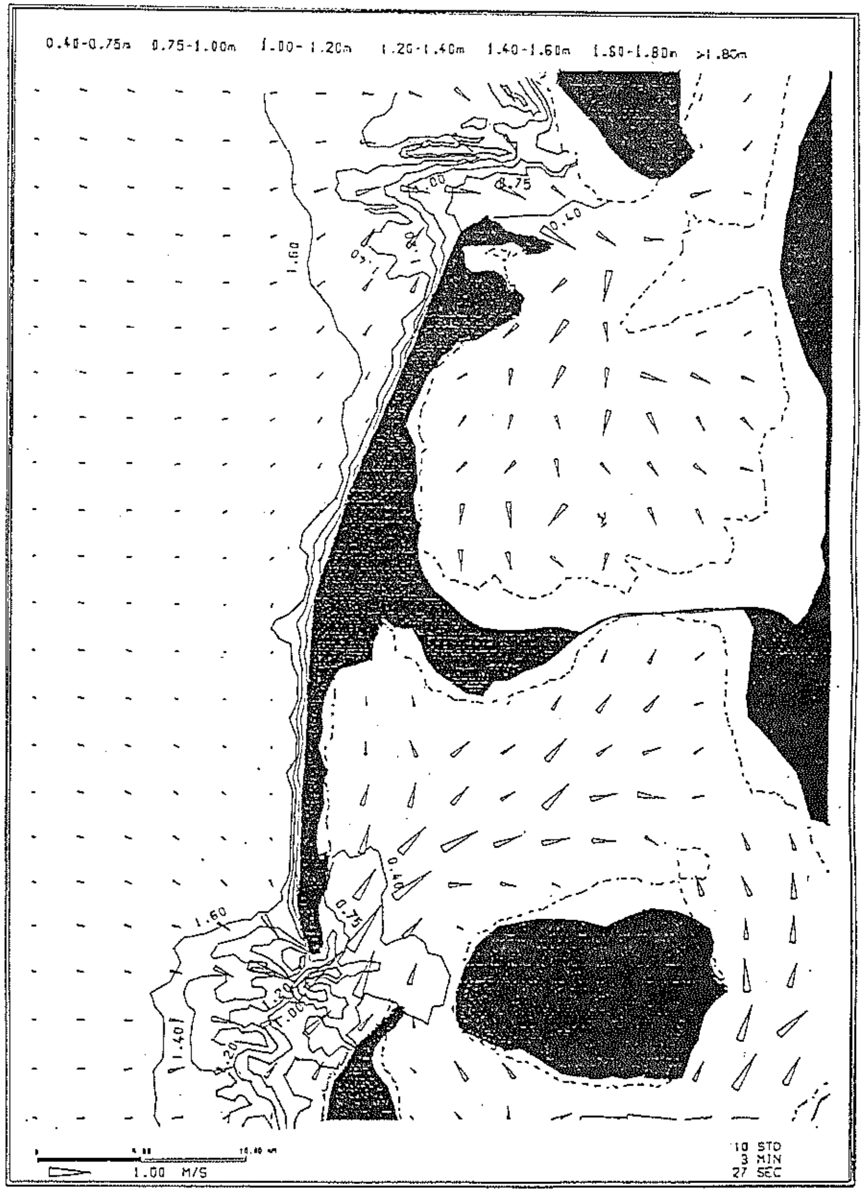

Figure 13: Wave height distribution and current velocity during flood phase

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