Propagation of a Tsunami-wave in the North Sea

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Abstract: The Tsunami disaster in Asia from December 26, 2004 was a reason to reconsider the risk of a Tsunami in the North Sea and the risk of a Tsunami impact to the German coast. Therefore, numerical simulations were performed to study the propagation of a Tsunami wave in the North Sea and find out the most affected areas in the North Sea and in the German bight. It could be shown that the German bight is well protected against a Tsunami wave by the shallow water depths of the wadden seas. This is not the case for the Dutch coast and the British Island which would be significantly affected by a Tsunami. A Tsunami entering the North Sea from the English Channel will not have any severe consequences in the North Sea, since this wave will be reflected and dampened in the English Channel.

1 Introduction and Motivation

The earthquake of December 26, 2004 in the vicinity of the island of Sumatra and the resulting tsunami wave resulted in one of the major natural disasters of the last centuries. The majority of the Tsunamis is generated by seaquakes, but Tsunamis can be also generated by volcano eruptions or flank failures due to slope instabilities. Examples are the eruption of the Krakatau in 1883 and the Storegga-sliding about 6000 B.C.. The probability of seaquakes is lower in the Atlantic ocean than in the Asian region due to the continental drift. Nevertheless, heavy Tsunamis also occurred in the Atlantic ocean. An example is the destruction of Lisbon in 1755.

Thus, the generation of a Tsunami in the Atlantic Ocean and its propagation in the North Sea is possible. No experience is available concerning the propagation of a Tsunami wave in the North Sea and the impact of a Tsunami wave to the coasts and estuaries of the North Sea. Hence, the objective of the present study is the propagation of a Tsunami wave in the North Sea.

Different hydrodynamic numerical models were applied to study the propagation of a Tsunami wave in the North Sea. The Tsunami wave at the seaward boundary was calculated analytically from the solitary wave theory. Then, the propagation of the Tsunami wave was investigated for three different scenarios:

- Without tidal influence
- With tidal influence
- For storm surge conditions

After that, the different scenarios were compared and the impact of a Tsunami to the German coast was determined from the maximum wave height.

Finally, conclusions for future research were derived on the basis of the findings in this paper.

2 Tsunami Wave Theory

Tsunamis generated by seaquakes are long periodic waves in between wind waves and tidal waves (Fig. 1). A tsunami can have a wavelength exceeding 100 km and a period in the order of one hour. Because a Tsunami has such a long wavelength, a Tsunami is a shallow-water wave. A wave becomes a shallow-water wave when the ratio between the water depth and its wave length gets very small ($d/L < 0.05$). The relative water depth for the Tsunami studied below is about $d/L = 0.02$. Shallow-water waves move with a speed equal to the square root of the product of the acceleration of gravity and the water depth. Therefore, Tsunami waves can be computed by the shallow water equations.

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In free surface flow in relatively thin layers, the horizontal velocities are of primary importance and the problem can be reasonably approximated as a quasi 2-dimensional system. The resulting equations, called shallow-water equations, include an equation for the free-surface elevation in addition to the horizontal velocities, and can be written in the same conservational form as the 2D Euler equations for isothermal compressible flow where the depth of the sea is equivalent to gas density in Euler equations. Therefore, utilities and experiences obtained from studies in 2D Euler flow can be applied to the simulation of tsunamis.

- Singular impulsive disturbance which displaces the water column
- Vertical change is normally caused by seismic events, underwater landslides or a combination of the two Shallow water waves with long periods and wave lengths: Due to the fact that the water depth of the ocean is shallow in comparison with the wave length

Tsunamis are governed by the shallow water wave equations since the ratio of $H/d \ll 1$ and $d/L \ll 1$, which means that nonlinear effects can be neglected. The Ursell-parameter U_r,

$$U_r = \frac{(H/L)}{(d/L)^3}$$

describing the nonlinearity of the waves, is in the order of U_r=400.

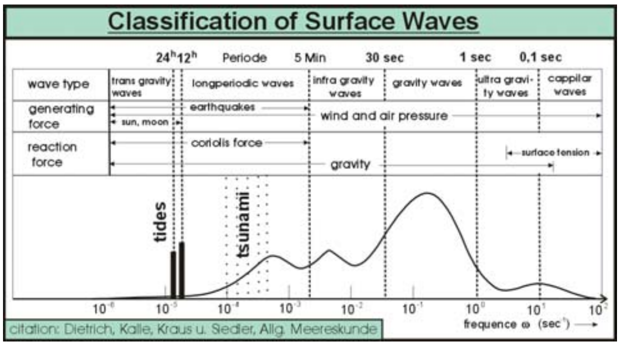


Fig. 1. Surface Waves

The Tsunami wave propagating over the continental shelf towards the shoreline is transformed mainly by shoaling, refraction and reflection. In this region, nonlinearities can not be neglected anymore and the fully nonlinear shallow water equations must be applied to solve the problem. The Ursell-Parameter increases rapidly in shallow water for Tsunami-waves ($Ur \approx 10^6$). Wave shoaling is the process resulting in a significant increase of the wave heights. The second effect which results in a significant increase of the wave heights of a Tsunami is the bay effect, which explains why many severe damages occurred in bays after previous tsunamis.

(a) Shoaling

The shoaling effect is shown in Fig. 2 according to linear wave theory (remark: linear wave theory is not valid for Tsunami waves, but the main processes can be approximated by linear wave theory as a first guess). The wave height of a Tsunami wave travelling into shallow water is increasing by wave shoaling following a relationship $H_x \sim 1/d_x^{1/4}$. In addition the wave length is decreasing resulting in a steeper wave. Thus, shoaling is an important effect to be considered.

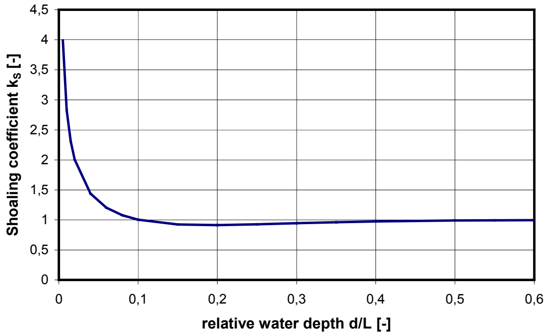


Fig. 2. Shoaling (according to Linear Wave Theory)

(b) Bay effect

The bay effect can be calculated by Green's law (1837):

$$\frac{H_x}{H_0} = \sqrt[2]{\frac{b_0}{b_x}} \sqrt[4]{\frac{d_0}{d_x}}$$

with: H = wave height, b = width of the bay, d = water depth, index 0 = offshore, index x = onshore

Thus, the wave height at position x is increasing with decreasing bay width and decreasing water depth. Green's law assumes no energy losses, no reflection and small wave heights. Fig. 3 shows the dependency of the relative wave height (H_x/H_0) from the relative bay width (b_0/b_x) and the relative water depth (d_0/d_x). Thus, it can be concluded that the wave evolution follows the laws $H_x \sim 1/d_x^{1/4}$ and $H_x \sim 1/b_x^{1/2}$. This was confirmed experimentally by Synolakis and Skjelbreia (1993) for solitary waves using different laboratory data.

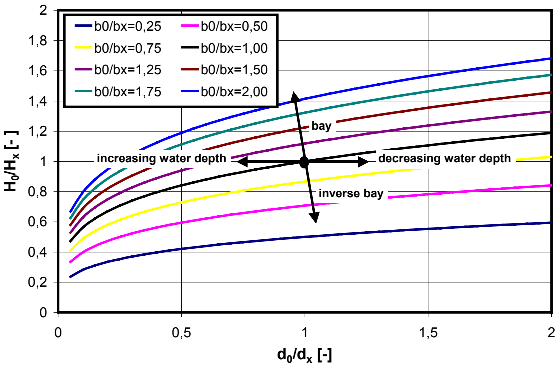


Fig. 3. Bay effect (calculated by Green's law)

In addition to the processes describe above, the Tsunami wave is transformed by wave refraction, wave diffraction, wave breaking and energy losses due to bottom friction and wave breaking. These processes will not be discussed in detail in the present study.

(c) Wave Run-up

Finally, a solitary wave hits the shoreline and inundates the hinterland. This is the process which causes the disaster. Several attempts were made to describe the run-up process by using Boussinesq-equations (Strybny, 2004) or the nonlinear shallow-water equations (Carrier and Greenspan, 1958;Tuck and Hwang, 1972; Spielvogel, 1976; Synolakis, 1987). Engineering equations to describe the run-up process were derived by Synolakis (1987) and Li and Raichlen (2001) for solitary waves.

$$\frac{R}{h_0} = 2{,}831\sqrt{\cot\beta}\left(\frac{H}{h_0}\right)^{5/4} + 0{,}104\cot\beta\,\frac{H}{h_0}$$

with: H = solitary wave height; h_0 = water depth at the toe of the slope; β = slope

Numerical calculations of wave run-up and wave run-down for solitary waves were performed by Lin et al. (1999) by using a RANS-model and by Strybny (2004) by using a Boussinesq-model.

Implications for a numerical model

A model grid is required which resolves the solitary wave by at least 5 to 7 points. By assuming a wave length of about 15 km in deep water (d = 300m) a grid resolution of about 3 km is required. The maximum grid size of the applied model is 14 km in the outer part of the North Sea. To approximate the given tsunami wave the grid spacing must be refined in a next step for the solitary waves which are not included in the present study yet.

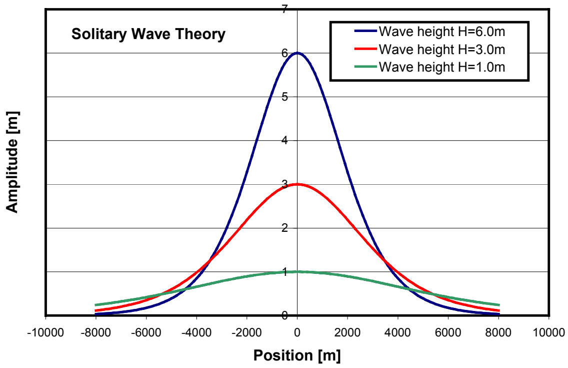


Fig. 4. Wave length of a solitary wave for d = 300m

3 Model Set-up and Boundary Control

3.1 Model Area and Model Grid

The set up of a hydrodynamical numerical (HN)-model of the North Sea and the German Bight including the German estuaries demands for

- a great spatial expansion of the model domain (the entire area of the North Sea),
- a relatively great resolution of the bathymetry within the coastal area and the estuaries and simultaneously
- a great number of calculation nodes.

This requirements demands for a high flexibility and variability of the computational grid and so only an unstructured triangular grid topology can be taken into account here.

The model domain of the North Sea model covers the whole area between Scotland/England and Norway/Denmark/Germany in west/east direction and north/south between a line near Fair Island/Stavanger and Plymouth/Ile de Batz in the English channel (Fig. 5). The bathymetry of the North Sea was provided by the British Oceanographic Data Centre (BODC) and by the German Local Authorities for the German Bight and the German Estuaries.

In its present state the model consists out of 111,000 nodes with a grid spacing between 14 km in the outer North Sea and about 200 to 600 m in the German Bight (for details see: http://www.baw.de/vip/abteilungen/wbk/Publikationen/fachz/NSEE_Modell_BAW-DH.pdf).

The same model grid and model area were used for all calculations described below.

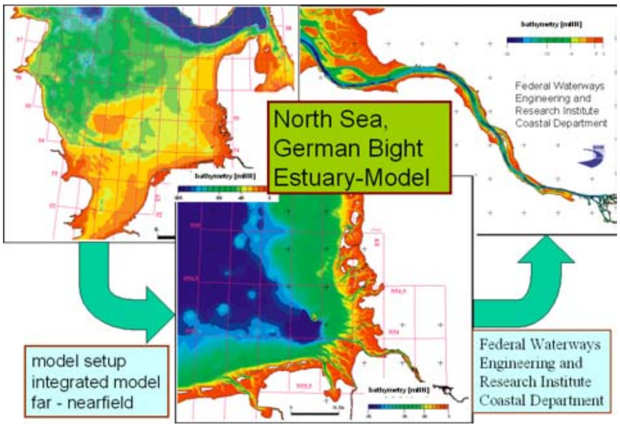


Fig. 5. The entire model area with detailed bathymetry reproduction in the German Bight and the Elbe estuary

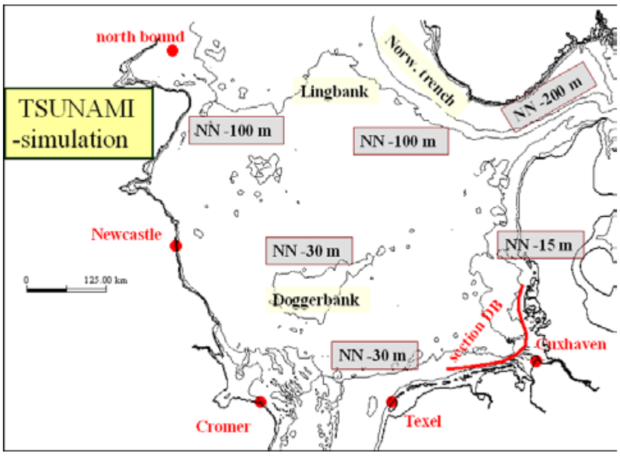


Fig. 6. Model Area

3.2 Boundary Control

The propagation of a fictitious Tsunami was investigated in the present study. Hence, variations of the wave height and the wave form were investigated (Tab. 1).

The so-called "geometrical wave" was generated first. This wave is shown in Fig. 7 and represents a sinusoidal wave with a small set-down in front of the wave. The "geometrical wave" has a given wave height and wave period.

In a second step, the boundary conditions were generated according to solitary wave theory for an average water depth of 300m at the model boundary. These solitary waves are shown in Fig. 8. The solitary wave was calculated according to the formulations by Dean and Dalrymple (1991) based on the shallow water wave theory developed by Korteweg and DeVries in 1895:

$$\eta = H \operatorname{sec} h^2 \left(\sqrt{\frac{3 \cdot H}{4 \cdot d^3}} \cdot c \cdot t \right)$$

with: η = surface elevation
H = wave height
d = water depth
t = time
c = wave celerity = $\sqrt{g(H+d)}$

Tab. 1. Investigated Boundary Conditions

	Wave height H		
	1.0 m	**3.0 m**	**6.0 m**
Geometrical Wave	-	√	√
Solitary Wave	In progress	In progress	In progress

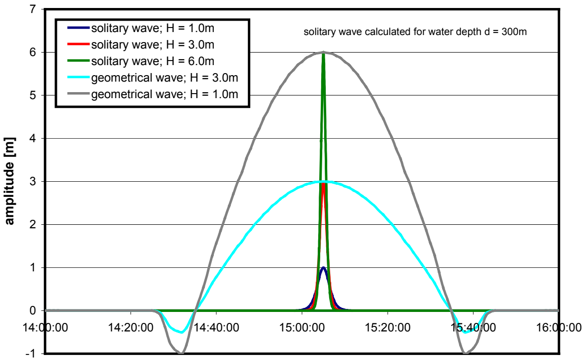


Fig. 7. Geometrical Wave in comparison to solitary wave

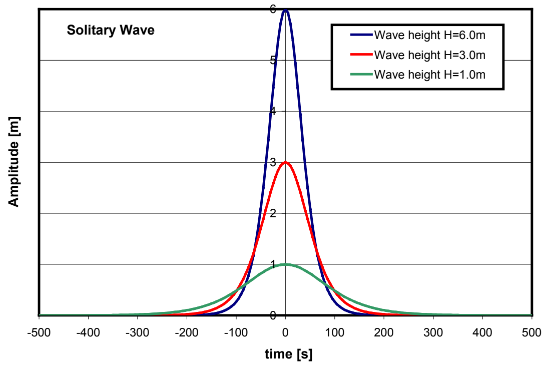
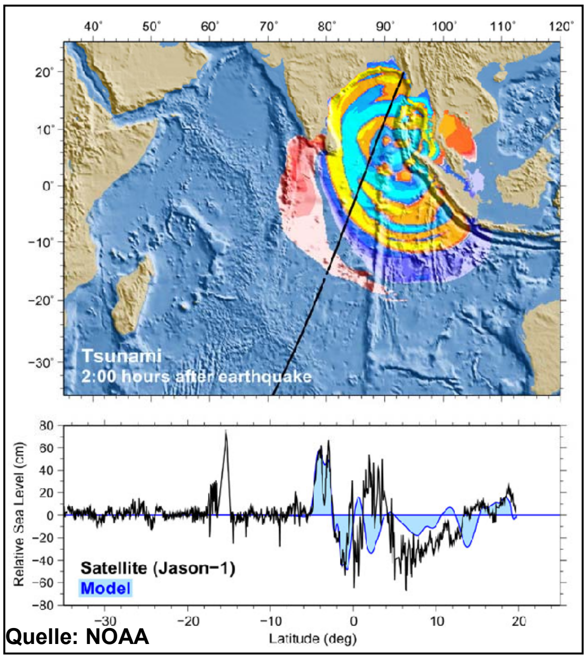


Fig. 8. Solitary Wave

The amplitudes of these scenarios are in the range of the measured amplitudes of the Sumatra Tsunami in Fig. 9 and Fig. 10. The tidal elevations of the Sumatra tsunami were determined from satellite data (NOAA) about 2 hours after the seaquake. The tidal elevations in the Indian Ocean were about <1m (Fig. 9). Nevertheless, it has to be mentioned that wave heights were measured nearshore for the Sumatra Tsunami including the shoaling effect higher than 1m. Thus, a wave height in between 1m and 2m is a realistic estimation at the continental shelf while higher waves are likely to occur nearshore. Therefore, the boundary conditions were varied in between H=1m and H=6m which is assumed to be a realistic range. The periods of the fictitious Tsunami in the North Sea are in good agreement to the observed periods of the Sumatra Tsunami (Fig. 10) which are in the order of 15 to 20 min at Male (Maldives).



Quelle: NOAA

Fig. 9. Satellite data from Sumatra Tsunami

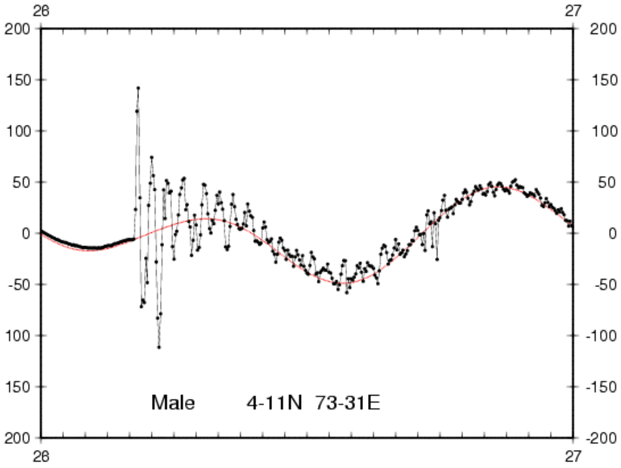


Fig. 10. Water level measurements at Male, December 26,2004 (source: http://ilikai.soest.hawaii.edu/uhslc/iotd/)

In addition, a Tsunami was also generated at the English channel. First investigations have shown that the water level elevations in the German bight are not significantly influenced by this Tsunami due to wave damping and wave reflection in the English Channel. Hence, this scenario is not studied here anymore.

3.3 Short description of applied numerical models

(a) Untrim

Untrim is a Finite Element / Finite Volume model which solves the 2D/3D Reynolds-averaged Navier Stokes (RANS) equations for an unstructured orthogonal grid. Untrim was developed by Casulli (Casulli and Walters, 2000) and a validation document is available under http://www.baw.de.

Untrim can be used to describe the following processes within a water body:

- propagation of long and short waves at the free surface,
- advective transport,
- horizontal and vertical turbulent diffusion,
- hydrostatic and non-hydrostatic pressures,
- Coriolis acceleration,
- horizontal and vertical gradients of density,
- bottom and wind friction

Thus, Untrim can be applied to the present problem. In addition, Untrim can be used for transport calculations (sediment, heat, salt), which was not required for the present study.

(b) Martin

The Martin simulation system is a powerful integrated public domain modeling tool for use in the field of free-surface flows.

Martin-Current2d is used to simulate free-surface flows in two dimensions of horizontal space. At each point of the mesh, the program calculates the water elevation and the two velocity components.

Martin-Current2d takes into account the following phenomena:

- Propagation of long waves, taking into account non-linear effects
- Bed friction
- Influence of Coriolis force
- Influence of meteorological factors: atmospheric pressure and wind
- Turbulence

- Influence of horizontal temperature or salinity gradients on density
- Dry areas in the computational domain: intertidal flats and flood plains

Martin-Current2d solves the transient shallow water equations using a high-precision stabilized finite-element method on an unstructured grid of triangular elements. The controlling of the internal time steps takes place automatically and as a function of the selected time integration schema. The implemented dry falling strategy is high volume conserving.

The model is implemented in AnsiC and a paralleled version of the software is also available using OpenMP. A detailed description of the code is available at: http://www.bauinf.uni-hannover.de/~milbradt/Martin/Current/current.html.

(c) Telemac

The mathematical model TELEMAC-2D is based on the finite element approach. TELEMAC-2D was designed to solve several depth-integrated transient non-linear partial differential equations (pde's). The model is actually used to study one ore more of the following physical processes:

- transport of water (conservation of the water mass)
- transport of linear momentum (conservation of linear momentum)
- transport of turbulent kinetic energy and turbulent kinetic energy dissipation (conservation of the turbulent kinetic energy)
- transport of salinity (conservation of the dissolved salt mass)

TELEMAC-2D was developed by *Laboratoire Nationale d'Hydraulique* from *Electricté de France, Direction des Etudes et Recherches* (EDF-DER), Chatou-Paris.

A detailed description of the code is available at:

http://www.baw.de/vip/en/departments/department_k/methods/hnm/mac2d/mac2d-en.html

4 Model Results

4.1 Tsunami Propagation without Tidal Influence

The propagation of an incoming Tsunami at the northern boundary of the North Sea between Wick (Scotland) and Stavanger (Norway) with a height of about 6 m (time t = 4.0 h) was investigated (Fig. 11).

The wave celerity $c = \sqrt{g \cdot d}$ in the Norwegian canal with water depths of about d = 200 to 300 m is much higher than in the central part of the North Sea with water depths in between d = 100 to 120 m. This results in a spreading of the incoming Tsunami wave including a decrease of the wave height (time t = 5.5 to 7.0 h).

This wave is now propagating along the Danish coast towards the German bight. The first Tsunami wave hits the German coast about 7 hours after arrival in the North Sea (time t = 11.5 to 13.0 h).

Regions with low water depths like the Lingbank (d = 60 to 70m) or the Doggerbank (d = 20 to 30m) result in a refraction of the Tsunami wave and an increase of the wave height.

Thus, the wave height is increased when propagating over the Doggerbank and the resulting wave hits the Northern entrance of the English channel.

The Tsunami wave hitting the English coast is partially reflected and propagates as a second wave from West to East. This second wave enters the German bight at time t = 8.5 to 16.0 h.

The reflection effect can be observed perfectly at time 8.5 h east of the Firth of Forth. At this time, the second wave train is generated which travels towards the German Bight. In addition the bay effect is obvious in the Firth of Forth at the same time.

Finally, some wave disturbance in the entire North Sea is remarkable, which was generated by reflection effects (Fig. 12).

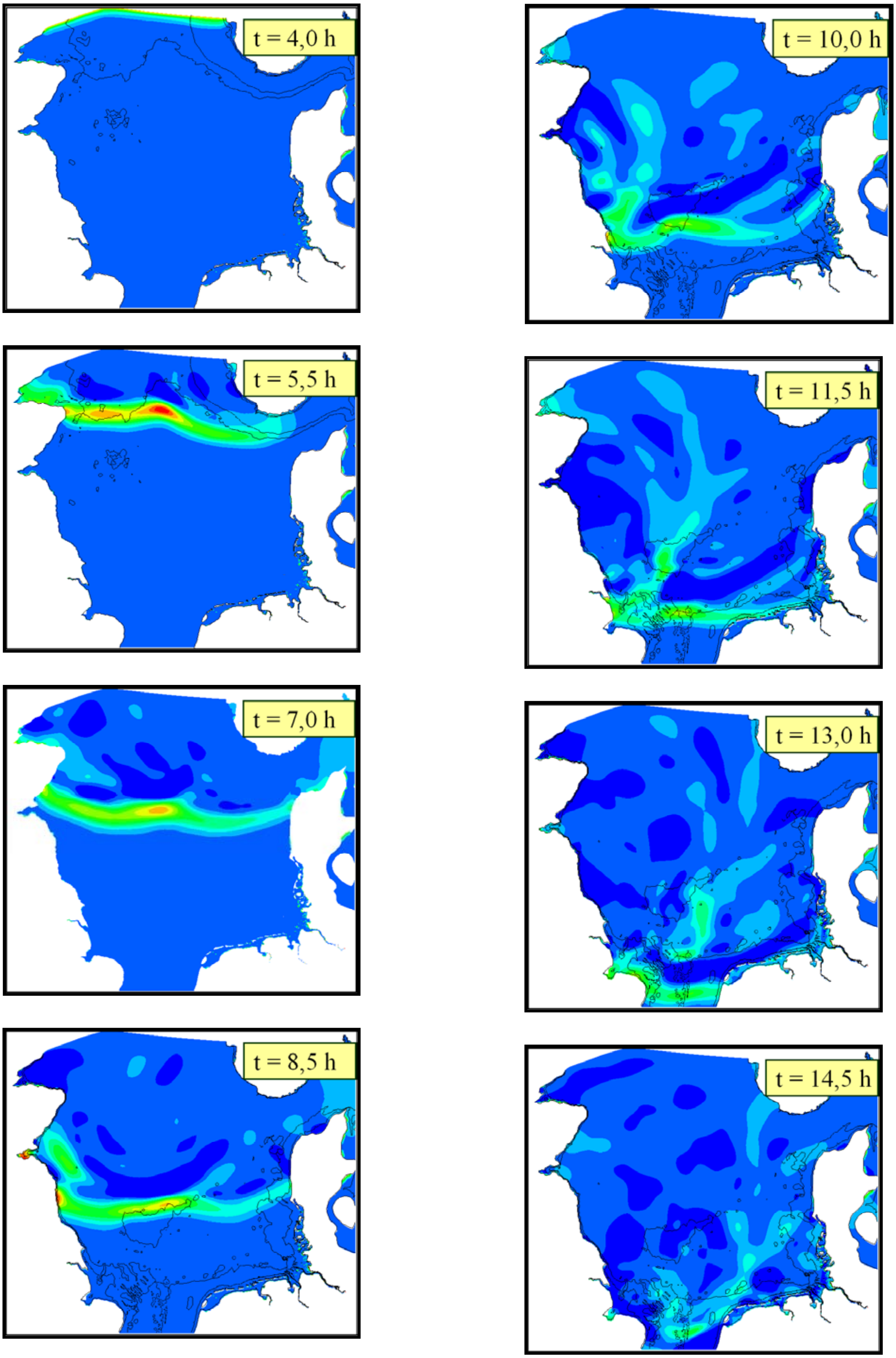


Fig. 11. Propagation of a Tsunami in the North Sea

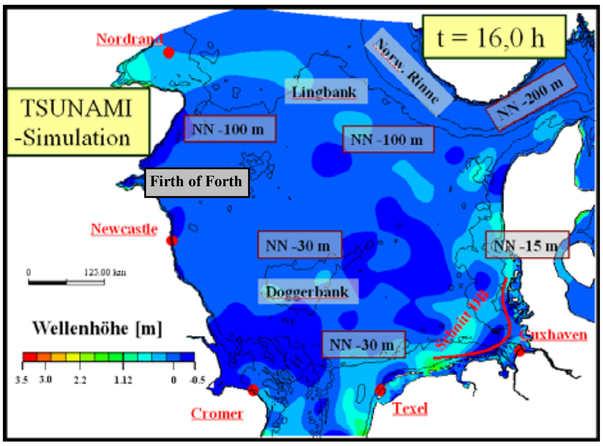


Fig. 12. Wave Disturbance in the North Sea after passage of a Tsunami

The water levels along the English coast show a double maximum from Newcastle which occurs about 5 hours after arrival of the Tsunami in the North Sea. The double peaks arrive at the English channel and at the Dutch coast with a time difference of about 2 to 3 hours. The time difference between the double peaks increases in the German Bight (Cuxhaven) up to 6 hours (Fig. 13).

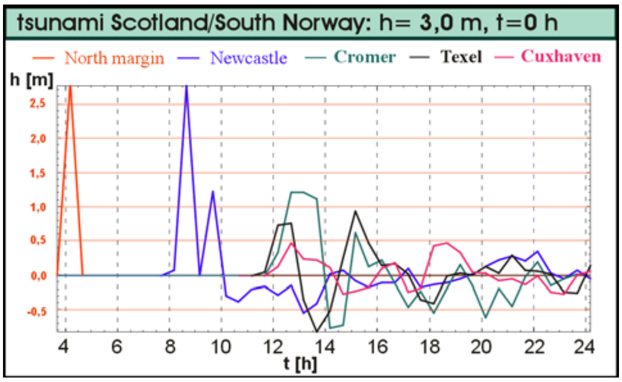


Fig. 13. Deformation of a Tsunami Wave in the North Sea

The wave height of the Tsunami decreases in the German bight significantly to about 1/6 of the wave height at the Northern boundary of the North Sea. A damping of the double peaks is obvious (Fig. 14).

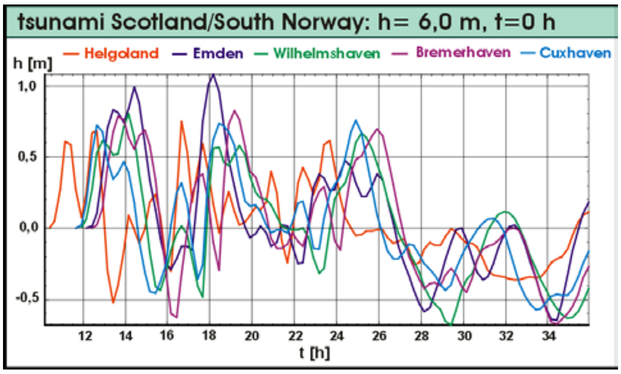


Fig. 14. Tsunami at special locations in the German bight

The distribution of the wave height maximums and minima along the German North Sea coast is very non-uniform.

The lowest wave heights were found in between the Elbe estuary and Eiderstedt. The highest wave heights were found along the east-frisian island and in the neighbourhood of Sylt (Fig. 15).

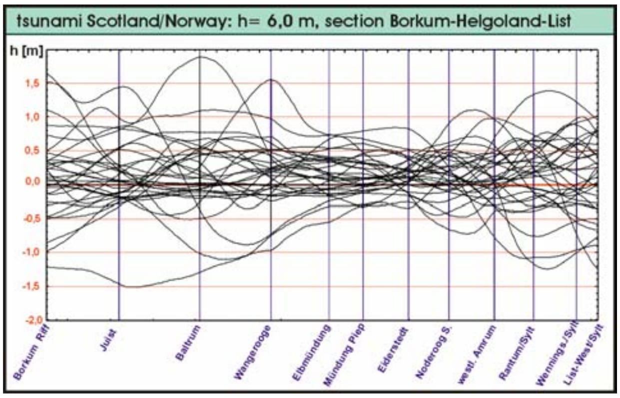


Fig. 15. Wave heights along the German coast

The Tsunami propagates in an estuary like a tidal bore up to the tidal limit without a remarkable decrease of the wave heights. Several individual waves are generated in the estuaries by reflection in the order of the first wave height. Three maximums were found at Zollenspieker (Fig. 16).

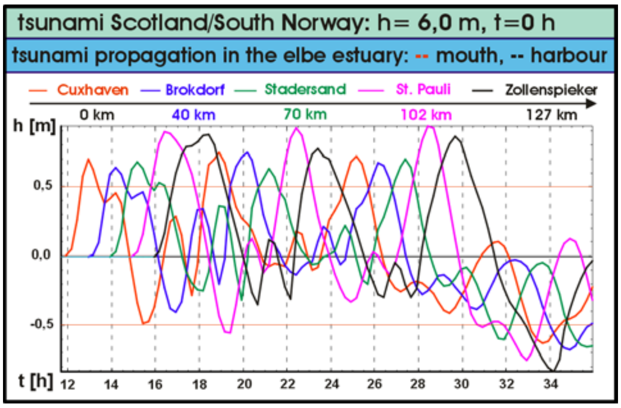


Fig. 16. Tsunami propagation in the Elbe estuary

Water level variations of up to 2 metres within 1 to 2 hours result in rapid fill and empty processes of the wadden areas and the tidal bays with high local current velocities (Fig. 17). Computed current velocities of about 1.0 m/s are in the same order of magnitude as observed current velocities during the Sumatra Tsunami (Scheffer, 2005).

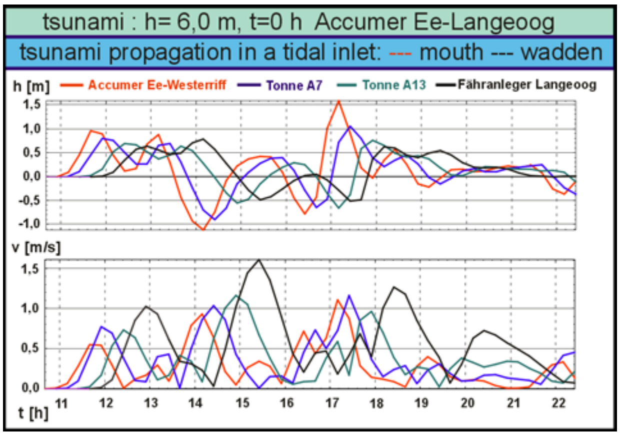


Fig. 17. Current velocities and water level variations

4.2 Tsunami Propagation with Tidal Influence

To investigate the influence of tidal motion on a tsunami wave propagating towards the German Bight, three model runs were set up:

- only tidal motion (tide only)
- tsunami wave input added to the tidal motion (tide + tsunami)
- only tsunami wave (tsunami only)

The difference between run 1 and 2 gives the tsunami wave propagation influenced by the tidal motion (tide + tsunami –tide). In relation to run 3 the effect of the tidal water levels and the currents can be evaluated (see Fig. 18).

It is obvious from Fig. 18 that the Tsunami wave is deformed at low tide because of the tidal current propagating against the Tsunami. Variations of the Tsunami wave at high tide are small compared to the variations at low tide.

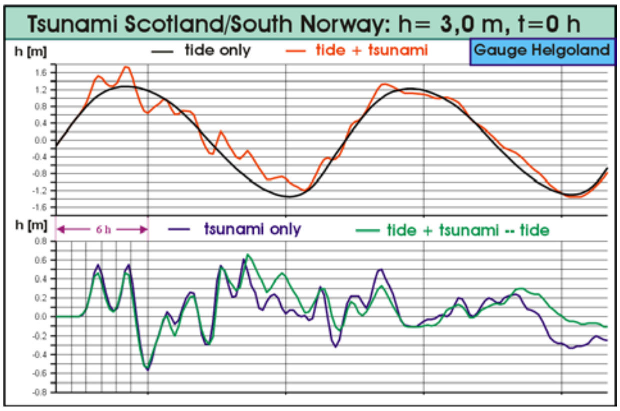


Fig. 18. Influence of tide on Tsunami propagation

4.3 Tsunami Propagation with different model approach

Three different model approaches were used to study the propagation of a Tsunami in the North Sea. The results are given in Fig. 19. The evolution of the Tsunami in the North Sea is comparable for the three model approaches. The phase is quite similar while the maximum height and the length of the Tsunami are slightly different. The differences are generated partly at the boundary because of the different boundary steering.

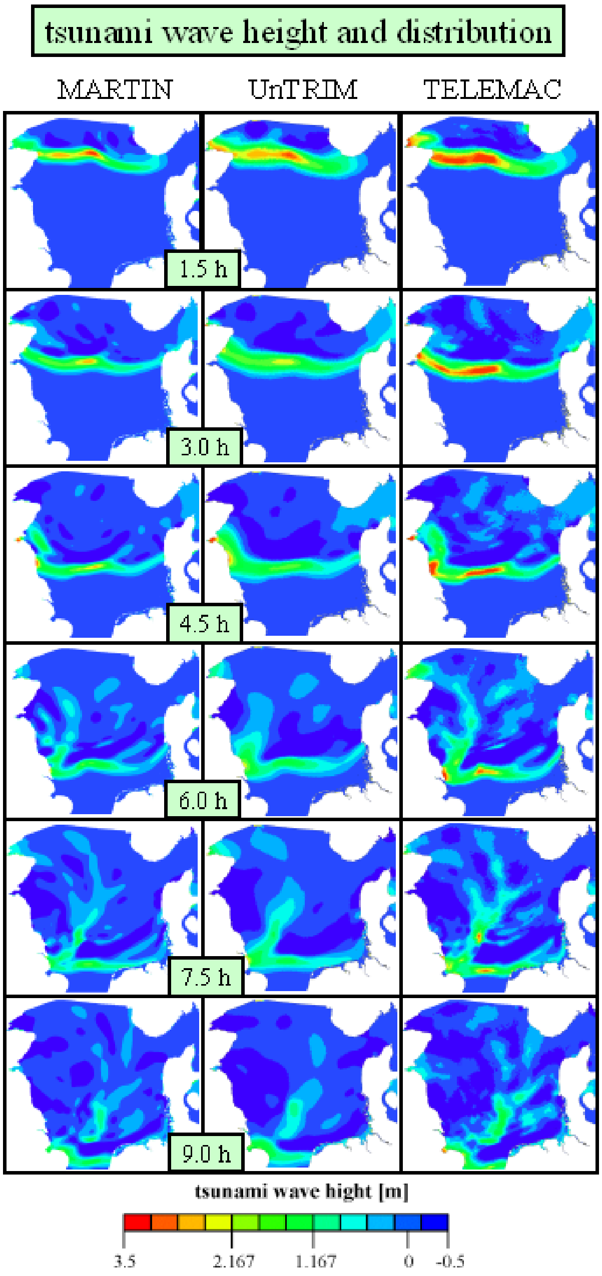


Fig. 19. Tsunami propagation with different model approach

5 Summary and Conclusions

The properties (boundaries, bathymetry) of the North Sea and the German bight are a good protection against Tsunami waves. This is not the case for the Dutch coast and the British Island. An incoming Tsunami wave at the northern boundary of the North Sea is transformed by shoaling, refraction, reflection and energy losses mainly by wave breaking. The amplitudes of the Tsunami wave at the German North Sea coast can be compared to the water level elevations of a storm surge. The governing processes are of course different.

Some aspects have not been studied in detail within the frame of this pilot project which are worth while to be studied in more detail within future research:

- Influence of different wave properties (wave height and period, wave direction) on the propagation of a Tsunami in the North Sea.
- Influence of a Tsunami on the flood protection system (dikes, dunes, seawalls) and the resulting risk of flooding.
- Influence of a Tsunami on other coastal structures (e.g. breakwaters, harbours, etc.)
- Risk assessment of a Tsunami for the German Coast.
- Possibilities and restrictions of numerical models, especially the momentum flux through the boundary..
- Extension of the model area across the continental shelf.

The present study represents the current status of the investigations. Further simulations and reflections concerning the propagation of a Tsunami in the North Sea are required and will be investigated in the near future.

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