Optimization of Beach Nourishment with Mathematical-Numerical Models

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Abstract:

This paper describes the components of numerical modelling the influence of beach nourishment, and the concept of an optimal design of such measurements. The mathematical-numerical model consists of three components of waves, currents and sediment transport (including beach change). The wave transformation module is based on a new hyperbolic formulation for monochromatic waves, the current module is horizontally two-dimensional, and the sediment transport describes suspended as well as bed loaded transport. In the second part of the paper we describe a systematical path to optimize the effects of beach nourishments. Starting from an ideal bathymetry we could reduce the space of parameters and studied the principal effects of some geometrical forms and locations. An example of an experimental beach nourishment shows the practicability of this path.

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INTRODUCTION

The beach is the boundary between the rough open sea and the land. The stability of beaches is important for the protection of human settlements, for the protection of high dunes and dikes serving as storm surge walls, and for the economy in touristic regions. Stability is supported by human interference in terms of artificial sand fills and groyne construction works. To optimize their effect and to avoid negative impacts, extensive inquiries are necessary. In the past few years numerical models have been acknowledged as an instrument entitled to the same rights as physical models or natural inquiries.

Starting from the necessities of an economic design and without negative impacts, a systematic research of different geometries and properties of beach nourishment using mathematical models is presented.

This paper describes the components of numerical modelling as well as the concept of a systematic and reproducible path to find useful geometrical forms of beach nourishments.

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BASIC EQUATIONS

The numerical simulation of coastal processes has a long tradition. Because the morphological change of the bathymetry near the beach is induced by the variable distribution of the surface waves as well as the nearshore currents, it is necessary to analyze the direction and height of waves, the direction and velocity of the nearshore current, and the turbulent energy. Most of the simulation systems describe the hydrodynamics near the beach based on a modular concept with an iterative indirect coupling of the various modules. In contrast, the numerical model presented here is based on the following compact system of time-dependent partial differential equations:

$$\frac{\partial K_i}{\partial t} = -\frac{\partial \sigma_a}{\partial x_i} + C_g \frac{K_j}{k}\left(\frac{\partial K_j}{\partial x_i} - \frac{\partial K_i}{\partial x_j}\right), \qquad \text{with} \qquad j \neq i = 1,2, \tag{1}$$

$$k^2 = K^2 - \delta^*, \tag{2}$$

$$\sigma_a = \sqrt{gk\frac{\tanh(kd) + s}{1 + s\tanh(kd)}} + \vec{K}\vec{U}, \tag{3}$$

$$\frac{\partial a}{\partial t} = -\frac{1}{2a}\frac{\partial}{\partial x_i}(U_i + C_{E_i})\,a^2 - \frac{S_{ij}}{\rho g a}\frac{\partial U_i}{\partial x_i} + \frac{U_i(T_i - T_i^B)}{\rho g a} + \frac{\epsilon_B}{\rho g a}, \tag{4}$$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial U_j(h+\eta)}{\partial x_j}, \tag{5}$$

$$\frac{\partial U_i}{\partial t} = -U_j\frac{\partial U_i}{\partial x_j} - g\frac{\partial \eta}{\partial x_i} - \frac{1}{\rho(h+\eta)}\frac{\partial S_{ij}}{\partial x_j} + \frac{1}{\rho\left(\bar{\xi} + h\right)}(T_i - T_i^B) \qquad \text{with} \qquad i = 1,2, \tag{6}$$

$$\frac{\partial h}{\partial t} = -\frac{\partial q_i}{\partial x_i}, \tag{7}$$

$$q_i = \int_{-h}^{0} U_i\, C\, dz + q_{b_i} \qquad \text{mit} \qquad i = 1,2, \tag{8}$$

$$\frac{\partial C}{\partial t} = -U_i\frac{\partial C}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\tau_i\frac{\partial C}{\partial x_i}\right) + S \tag{9}$$

We can say that the model consists of three submodels for calculating waves (1-4), nearshore currents (5-6), and sediment transport with bottom changes (7-9) which are fully coupled. The mathematical type of this system is a multi-dimensional advection-diffusion problem. This is of great importance rule for the numerical realization. A detail description of the components will be provided in the following subsections.

A Cartesian co-ordinate system is introduced in where the plan coordinate-axes for x and y can be placed arbitrarily. The z co-ordinate is positive upwards. Starting from the importance of the wave distribution in the surf zone near a coast we begin with the wave components.

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Wave Model

Waves approaching shallow water and the beach areas in the absence of currents are deformed due to shoaling, refraction, reflection, diffraction and breaking. The effect of reflection is negligibly small. Thus the theory of monochromatic waves may be used.

To describe the effects of interaction of waves and currents as well as the interaction with the changeable bathymetry a new hyperbolic type wave formulation has been introduced [3]. With this formulation we can use the nonlinear wave potential developed by YOO (see [4]) as well as the classical potential. The system of equations (1-4) contains the wave vector $\vec{K}$ and the calculation of the wave amplitude a. In these equations C_g stands for the wave group velocity and C_E for the energy transport velocity. The radian frequency σ is calculated by the dispersion relation (3), and the BATTJES relation (2) describes diffraction effects. The equations include the influence of bottom slope [4]. This enters the equations by the terms

$$s = -\frac{1}{k}\frac{1}{A}\frac{\partial A}{\partial x_i}\frac{\partial d}{\partial x_i} \qquad \text{and} \tag{10}$$

$$A = -\frac{g\,a}{\sigma_r}. \tag{11}$$

To describe the input of turbulent energy due to wave breaking in the water body, a lot of breaking criteria are realized. The wave energy loss is described by the energy dispersion coefficient ϵ_B. We use the D ALLY, DEAN and DALRYMPLE [1] model to assume that there is a stable wave height after breaking equal to some fraction of water depth. For wave breaking the rate of energy dissipation is proportional (with the factor K_B) to the difference between the actual and the stable wave energy flux ($(E\,C_g)_s$).

$$\epsilon_B = -\frac{K_B}{h}(EC_g - (EC_g)_s). \tag{12}$$

Alternatively a wave-front model (see [2]) can be used. This wave approximation can additionally describe the special effects of crossing waves and has a better approximation for diffraction. Additionally the wave-front approximation gives good intuitive pictures for wave-front distribution. An automatic refinement of computational points of the wave front with respect to the hydrodynamic requirements is calculated. The wave-front model cannot be integrated directly into the above model philosophy. The mathematical type of this model is a characteristic solution and can be coupled only iteratively with the equation of nearshore currents and sediment transport.

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Current Model

The tidal motion is simulated under the assumption of vertically integrated flow. The corresponding equations of momentum are (6) and the mass conservation is described by equation (7). In these equations $\vec{U}$ stands for the vertically integrated velocities, h for the water depth at rest, and η for the free surface. The earth acceleration is g, ρ is the water density, $\mathbf{T}$ represents turbulent effects, and $\mathbf{T}^B$ the bottom friction. Commonly the Taylor-Newton formulation is used, to which the influence of orbital motion of the waves is added.

The radiation stress term S_{ij} represents the momentum flow induced by the deformation of short period waves running at the surface of the water body. These effects are very small for deep water conditions, however, they are important in regions of deformation and breaking of short waves. These terms couple long and short wave propagation.

For simplicity, Coriolis, atmospheric pressure and wind terms are omitted.

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Sediment Transport and Bottom Change Model

Under breaking waves and due to high flow velocities, material is being picked up from the bed and transported in the flow regime. This transport can be distributed in bed and suspended load. To determing the concentration of the suspended sediment we use a vertically integrated advection-diffusion equation (9). In this equation C is the suspended sediment concentration, q_i is the transport rate in the x_i-direction as a sum of suspended and bed load $q_{b\,i}$. The diffusion coefficient is called with τ_i and S is a source or sunk term, respectively. The model assumes that there is a maximum possible concentration C_{max} and an actual concentration C. The term S is proportional to the difference between these two concentrations:

Source:

$$C_{maxs} > C \quad : S = \lambda_e\,(C_{max} - C) \tag{13}$$

Sunk:

$$C_{maxs} < C \quad : S = \lambda_s\,W_s\,(C_{max} - C) \tag{14}$$

with erosion and sedimentation parameters (λ_e and λ_s) and the sediment fall velocity W_s.

Calculating how much material enters and leaves a computational cell leads to the bed continuity equation (7). From this the evolution of bathymetry may be calculated. The term h stands for the variation of the water depth.

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NUMERICAL APPROXIMATION

The given system of time-dependent partial differential equations is solved by the finite element method. Triangular grids are used with linear interpolation functions in space for the state variables. Higher order terms are removed by partial integration. The integration follows the stream-line upwinding technique. The upwinding parameters are obtained proportional to the characteristic velocities of the different components by introducing corresponding element-PECLET-numbers.

The time integration is performed in the common step by step manner. Due to the simple structure of the software different 1st to 5th order time-integration schemes with time-step control are implemented.

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OPTIMIZATION OF THE GEOMETRICAL SHAPE OF BEACH NOURISHMENT

The subject of the presented inquiry were different measures in a system of sandbars and grooves of the island Sylt. For example tombolos, double tombolos and beach parallel beach nourishments in different locations and geometries were investigated. Because of the complexity of parameters necessary to optimize such beach nourishment we selected an iterative procedure.

In a first step the tidal flow and the propagation of short waves were analyzed for the global area around the island. This inquiry generated boundary conditions for small area studies. Figure 1 shows a wave field for waves under westerly wind. At the tips of the island complicated processes of shoaling, refraction, diffraction and breaking can be seen due to extended shallow banks.

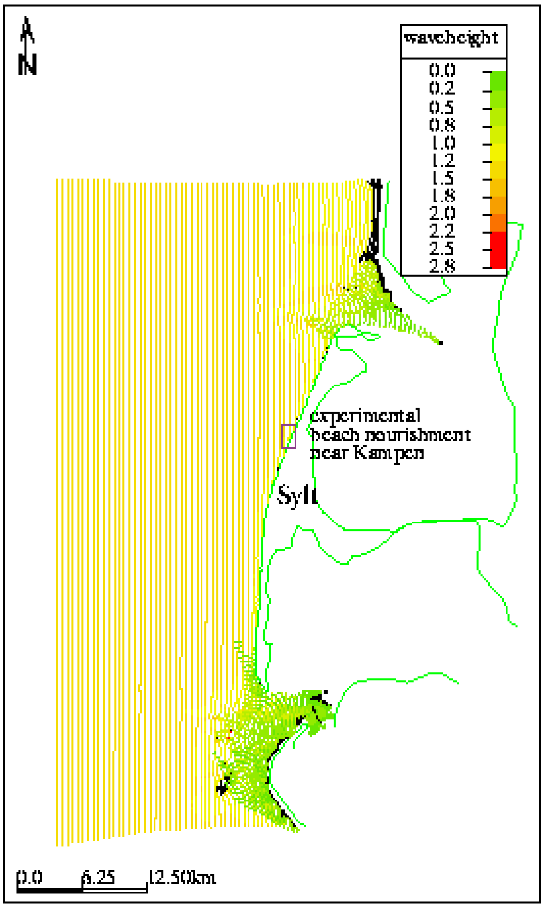


Figure 1: Large area wave simulation

Starting from the natural situation of a system of sandbars and grooves of the island Sylt an idealized bathymetry was used. Such a bathymetry allows a clear relation between cause and result of the physical processes. Figure 2 shows the calculated wave fields and velocities at one particular time of a tide for different locations of tombolos.

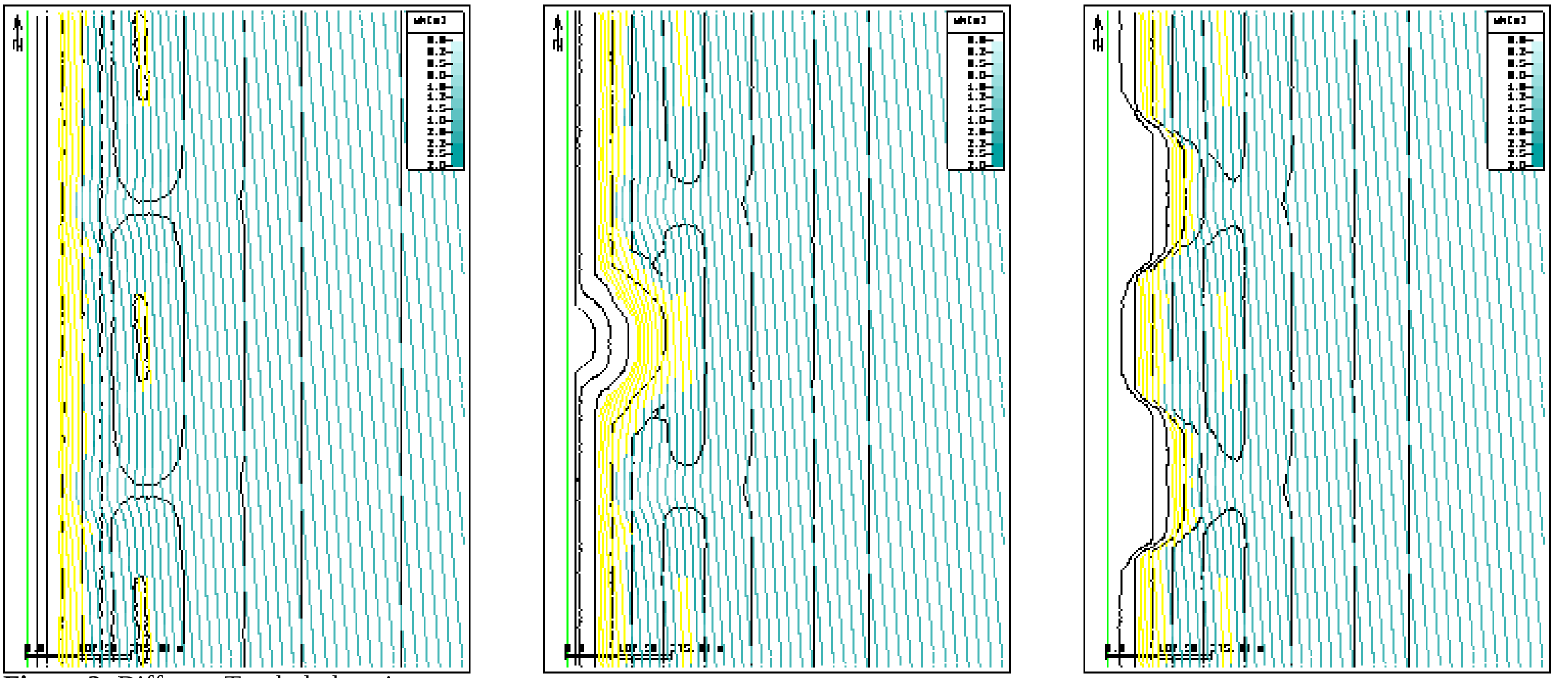
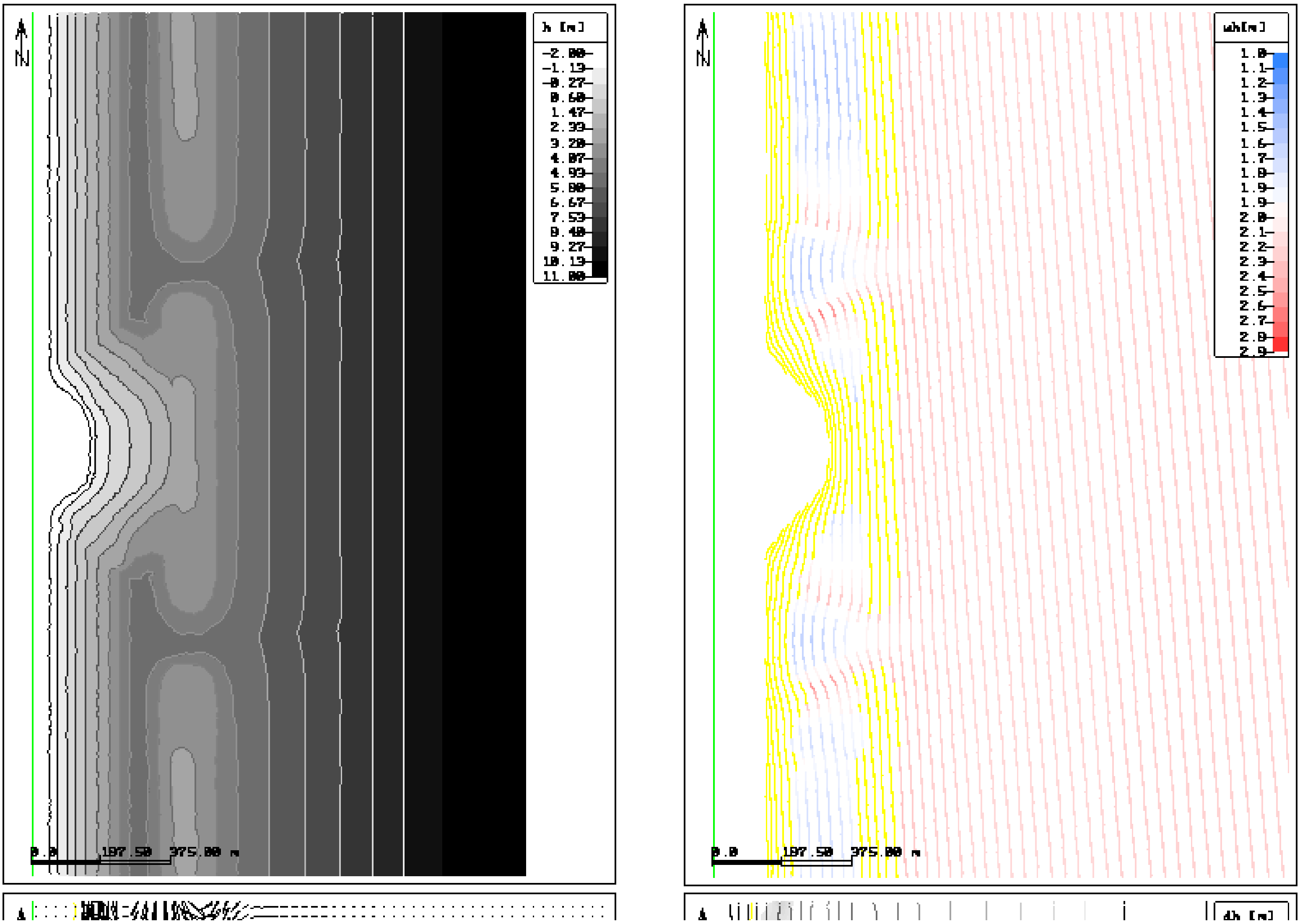


Figure 2: Different Tombolo locations

Applying the combined wave-current model the geometry was investigated in the first place to provide an energy dissipation as homogeneous as possibly at and in the vicinity of the beach nourishment corps, and to minimize the occurring streaming. We may now select a lot of parameters and locations for the beach nourishment, meeting the above criteria.

In a next step, with a smaller set of such measurments we can use the coast development model. Figure 3 shows a complete set of simulation results at one particular time of a tide. This figure provides the bathymetry for a sandfill reaching from the beach towards the underwater bar at a distance of about 400 meters from the beach. The flow field shows two typical eddies forming at the shoulders of the sandfill due to waves running nearly perpendicular to the coast. At last the areas of erosion are given for a simulation time of just one tide.



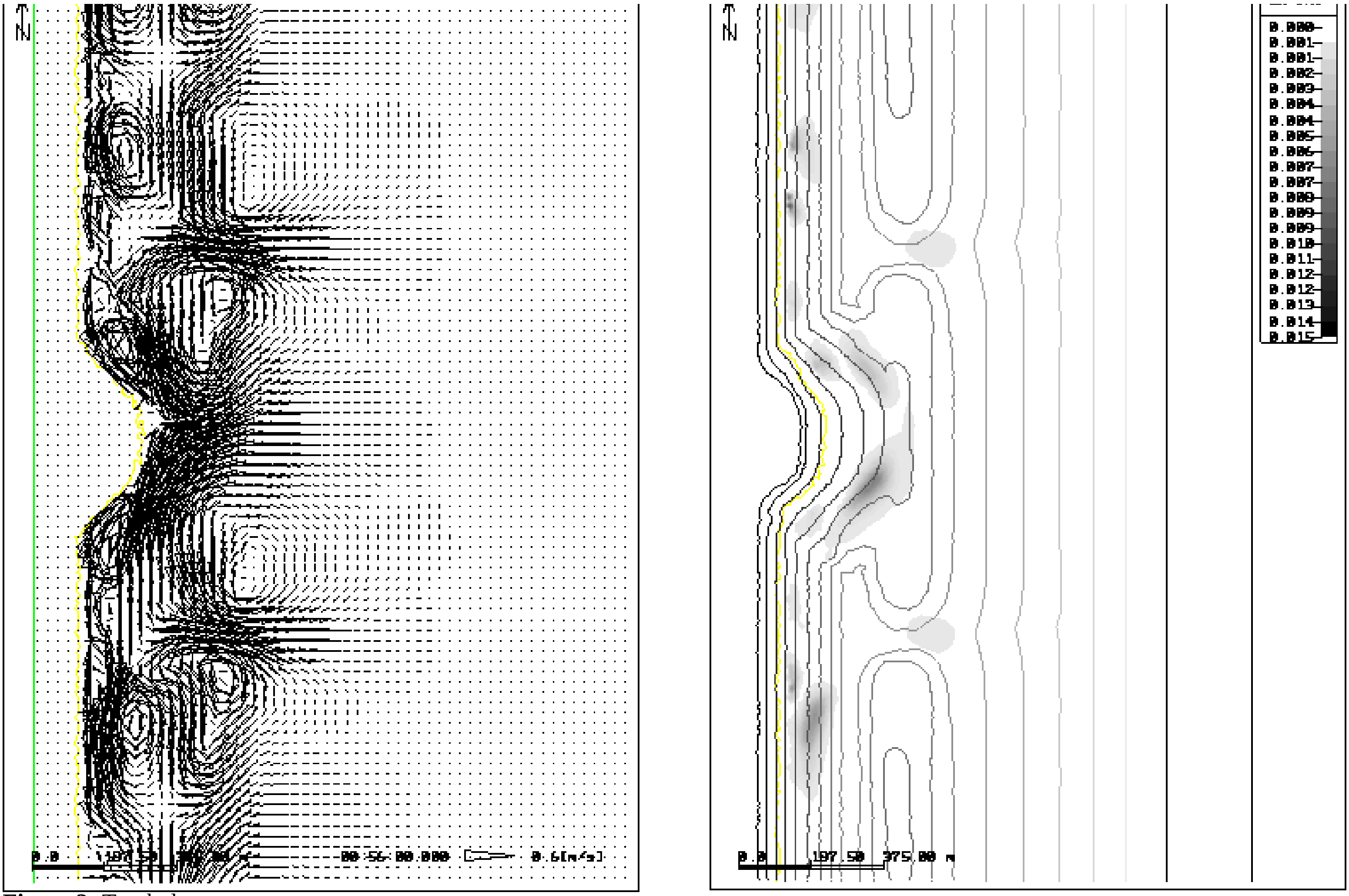


Figure 3: Tombolo

Using results of the idealized bathymetry and to verify the coast development model, wave-current-bottom-change simulations for a natural bathymetry were investigated. Supplementing these investigations measures at sandbars - such as elevation and broadening - were taken into account.

In addition an experimental beach nourishment near Kampen (Sylt) was performed verifying the results of the mathematical models. With the studies in the idealized bathymetry a lot of natural hydrodynamic effects are now plausible.

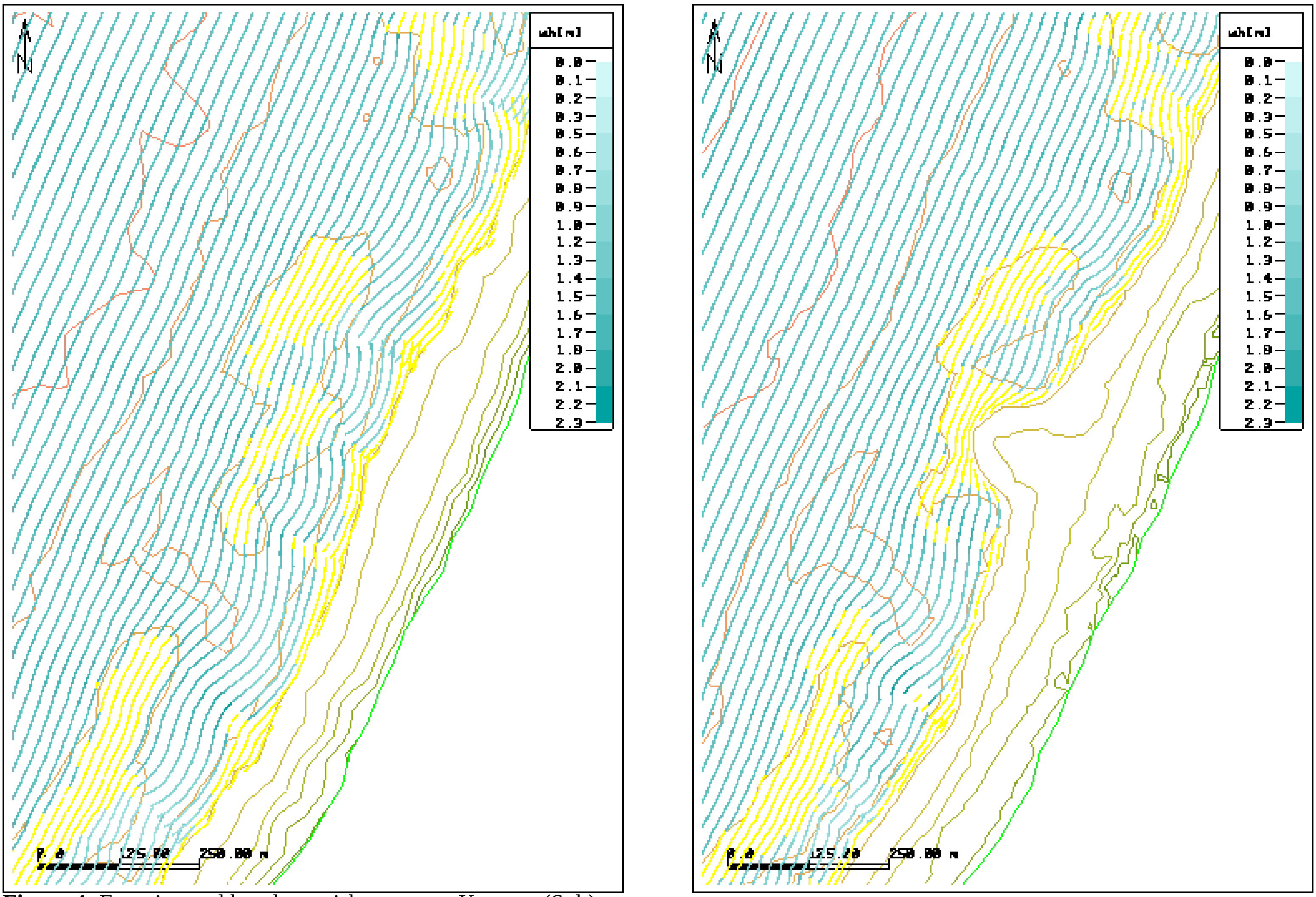


Figure 4: Experimental beach nourishment near Kampen (Sylt)

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CONCLUSION

It is impossible to provide an exact mapping of the natural processes in their full complexity, because the space of unknown physical parameters is substantial and a lot of processes are stochastical. However, in this contribution we present effective mathematical methods to give systematical and reproducible approximations to estimate the effects of beach nourishments.

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