Holistic modeling of morphodynamic processes with stabilized finite elements

Peter Milbradt

Institute of Computer Science in Civil Engineering, University of Hannover,
Am Kleinen Felde 30, 30167 Hannover, Germany
e-mail: milbradt@bauinf.uni-hannover.de

Abstract A holistic modeling approach of processes related to morphodynamic changes in coastal regions is presented. The model consists of a unified mathematical formulation for currents, waves and sediment transport processes. The single system of hyperbolic differential equations implies direct coupling of the different model components and is solved with stabilized finite triangular elements. A morphodynamic modeling application in a micro-tidal region of the Southern Baltic Sea is presented.

Keywords morphodynamic modeling, stabilized finite elements, Baltic Sea

Introduction

The hydro- and morphodynamic processes in the nearshore area create highly complex phenomena. Suitable modeling tools are necessary for the assessment of natural developments in the coastal zone as well as of impacts of human interventions in form of coastal protection buildings. This leads to new requirements for numerical models applied in coastal engineering. An isolated view of hydrodynamics and morphodynamics on the one hand and the different phenomena in the water body, like waves, currents, changes of salt concentration and temperature on the other hand are often not sufficient. For complex problems it is more and more necessary to interconnect different model approximations. In this paper I present a holistic model concept as well as its closed numeric approximation on the basis of stabilized finite elements.

Holistic approach

The dynamics of the three-dimensional water body can be described by the three-dimensional Navier Stokes equations. In order to resolve the dominant physical phenomena by an appropriate numeric simulation model, high spatial and temporal discretisations are necessary. For a conversion in numeric simulation models additional approximations are used. A substantial concept here is, following the Reynolds decomposition of currents, a decomposition in three parts: currents, waves and turbulent currents. Each of these subprocesses can be described again by a multiplicity of models published in the literature and implemented in available simulation packages. Further models describe the change of the salt concentration, the temperature and other constituents in the water. In analogy to hydrodynamics, morphologic changes are described by continuity and transport models. The mathematical formulation of these physical models leads generally to systems of time-dependent partial differential equations.

Efficient numeric simulation models were developed in the past for these partial models. The simulation of the interaction of different processes, like waves, currents and sediment transport is implemented with classical concepts via the coupling of existing process models.

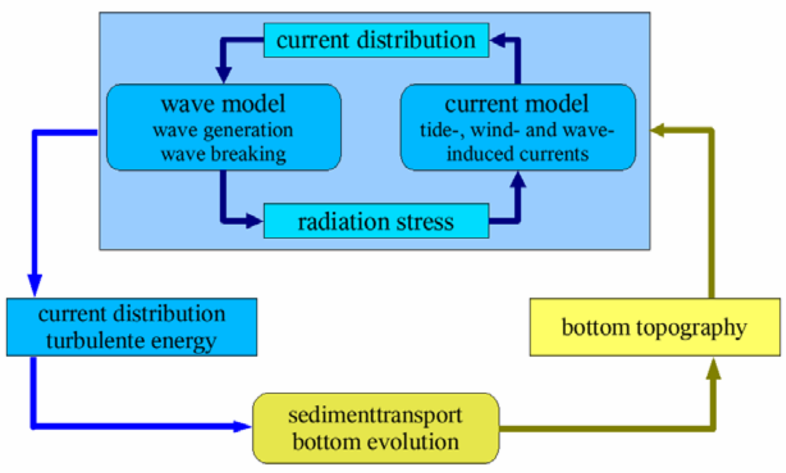


Figure 1 Classical model concept

This iterative coupling of models requires a lot of experience of the modeler with regard to the coupling time steps.

A recent development by Milbradt (1995) on the other hand, shows that all noted processes can be described in a closed set of equations, which can be solved directly.

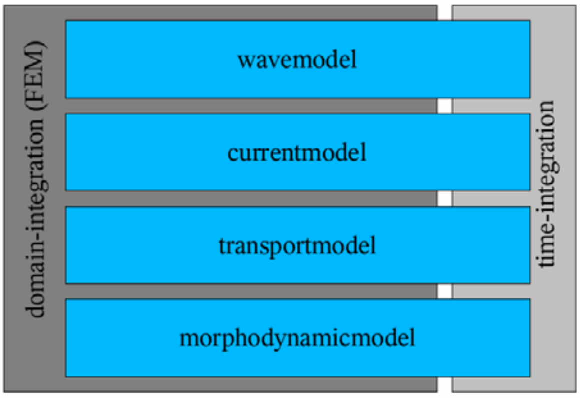


Figure 2 Holistic model concept

The numerical approximation of the physical processes now is directly coupled and can be solved with uniform numeric methods.

Stabilized finite element approximation

The majority of the equations of the hydro- and morphodynamic are transport equations. The used numeric procedures must reflect this character of the equations. Classical finite differences and finite elements procedures are not suitable for transport equations. Since Hughes and Brook (1982) suggested a stabilization, different stabilizing procedures were developed for the scalar linear transport equation.

As an example, the following system of 9 equations is used to simulate the morphodynamic processes.

waves

$$\frac{\partial K_i}{\partial t} = -\frac{\partial \sigma_a}{\partial x_i} + C_g \frac{K_j}{k}\left(\frac{\partial K_j}{\partial x_i} - \frac{\partial K_i}{\partial x_j}\right)$$

$$\frac{\partial \sigma}{\partial t} = -\left(U_i + C_{g_i}\right)\frac{\partial \sigma}{\partial x_i} - k_{x_i}\frac{\partial u_i}{\partial t} + f\frac{\partial h}{\partial t}$$

$$\frac{\partial a}{\partial t} = -\frac{1}{2a}\frac{\partial}{\partial x_i}\left(U_i + C_{E_i}\right)a^2 - \frac{S_{ij}}{\rho g a}\frac{\partial U_i}{\partial x_i} + \frac{U_i\left(T_i - T_i^B\right)}{\rho g a} + \frac{\varepsilon_B}{\rho g a}$$

currents

$$\frac{\partial \overline{\eta}}{\partial t} = -\frac{\partial U_j d}{\partial x_j}$$

$$\frac{\partial U_i}{\partial t} = -U_j\frac{\partial U_i}{\partial x_j} - g\frac{\partial \overline{\eta}}{\partial x_i} - \frac{1}{\rho d}\frac{\partial S_{ij}}{\partial x_i} + \frac{1}{\rho d}\left(T_i - T_i^B\right)$$

sediment-transport

$$\frac{\partial C}{\partial t} = -U_i\frac{\partial C}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\tau_i\frac{\partial C}{\partial x_i}\right) + S$$

$$q_i = \int_{-h}^{n} U_i C dz + q_b$$

$$\frac{\partial h}{\partial t} = -\frac{1}{1-n}\frac{\partial q_i}{\partial x_i}$$

Figure 3 System of equations

The numerical approximation is based on finite triangular elements and uses the semi-discrete SUPG. With the solution of the above instationary partial differential equation, a semi-discrete approximation is used, according to which first the time derivatives are taken as an independent unknown quantity.

Each of these partial models for waves (W), current (U) and transport (S) can be transformed into a simplified form by introduction of suitable differential operators:

$$L_v \equiv A_{vi}\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i}\left(K_{vij}\frac{\partial}{\partial x_j}\right)$$

Figure 4 Differential operators

$$\frac{\partial W}{\partial t} + L_W W + Q_W = 0$$
$$\frac{\partial U}{\partial t} + L_U U + Q_U = 0$$
$$\frac{\partial S}{\partial t} + L_S S + Q_S = 0$$

Figure 5 System in operator notation

Here the Q's are source and sink terms. Both the differential operators and the source and sink terms depend generally on all unknown quantities of the system. For the subsystems, a Predictor-Corrector implementation is realized, following the derivation of stabilized finite elements as combination of standard Galerkin method and least squares approximation:

$$\int_\Omega \phi_l \left([\phi_i]^T \left[\frac{\partial U^i}{\partial t} \right] + L\left([\phi_i]^T \left[U^i \right] \right) + Q \right) d\Omega$$
$$+ \sum_k \tau_k \int_{\Omega_k} L(\phi_l) \left([\phi_i]^T \left[\frac{\partial U^i{}_G}{\partial t} \right] + L\left([\phi_i]^T \left[U^i \right] \right) + Q \right) d\Omega_k = 0$$

Figure 6 Stabilized Finite Element Approximation

with ϕ_i shape functions, Ω the solution domain and Ω_k the domain of the Finit Element k.

The time derivative of the unknown quantities in the second line is determined by the Galerkin approximation in the first line. The procedure developed in such a way can be understood as a Predictor-Corrector procedure for the domain approximation. In the end we have an ordinary differential equation, which we can solve explicitly by different methods.
It is well-known that the so-called upwinding coefficient τ has a substantial influence on the quality of the approximation. For the determination of the upwind-coefficients of the subsystems the following generalisation was used.

$$\tau := \frac{\alpha_{opt} \cdot h}{2 \cdot \|A\|}$$

Here a suitable operator norm of the transport operator is used. On the basis of the general definition (Kolmogorov) of the norm of continuity operators in (Euclidean) normed spaces the following operator norm is used.

$$\|A\| := \sqrt{\sum_i \|A_i\|^2}$$

Where the norm of the operator component is calculated by

$$\|A_i\| := \left| \lambda_{\max}(A_i) \right|$$

and $\lambda_{\max}(A_i)$ is the absolutely largest eigenvalue of the matrix A_i.

The parameter α_{opt} is determined for each element by

$$\alpha_{opt} = \coth Pe - \frac{1}{Pe}$$

with the element Peclet number $$Pe = \frac{\|A\| \cdot h}{2 \cdot \|K\|}$$

Model application

Figure 7 Sand flat Bock, Southern Baltic Sea

The presented model system is used to evaluate the morphodynamic processes within the range of the sand-flat “Bock”. The German coastal regions of the southern Baltic Sea can be characterized as micro-tidal regimes where shorelines are formed by wind-generated waves and currents.

For a 10-days storm event in January 2000, a completely coupled simulation of currents, waves and morphodynamics has been carried out. A cascade of models was set up and validated. First of all, a large-scale low-resolution coastal model for the regional flow, wave and sediment transport simulation (see Figure 8) was established. All necessary boundary conditions for high-resolution local models for the study of near shore wave-current-transport interactions in any focal area of the domain are available from the validated coastal model.

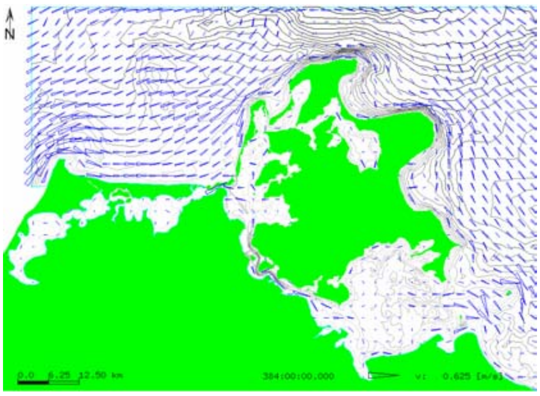
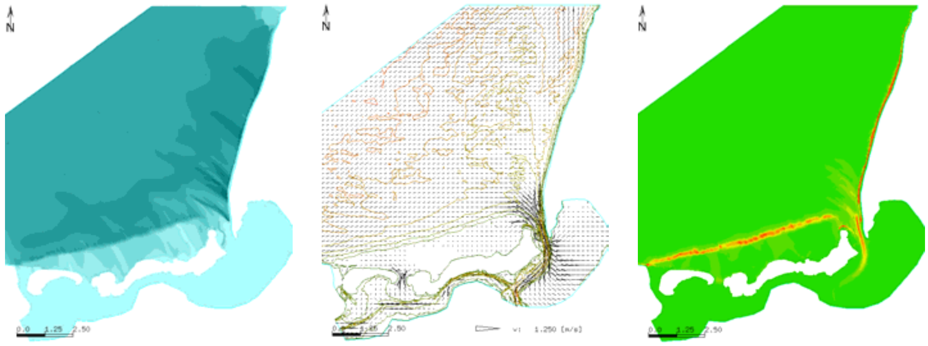


Figure 8 Large scale flow patterns in coastal model

Morphodynamic simulations of the storm event have been carried out with the local model (see Figures 9) where currents, waves and sediment transport are completely coupled, i.e. interacting at each time step. Estimations of the morphologically effective contributions related to the large-scale and the wind-induced currents as well as the swell-effect correspond to the measured tendencies of the morphological development in the area under investigation.



a) waves b) currents c) sediment transport

Figure 9 Local model

The analysis of the transport-capacities and -patterns of the areas of sediment accretion and erosion correspond well with the morphological structures known from sea surveys in the area under investigation. Particularly impressing are the indications of changes along the beach-ridge on the extended sand flat computed by the morphodynamic model as well as the development of sand-reefs during a storm-event (Figure 10) corresponding to patterns actually found in the field

The model results presented here are part of a model intercomparison, which is presented by Lehfeldt et al. (2002) at the ICCE2002 in greater detail.

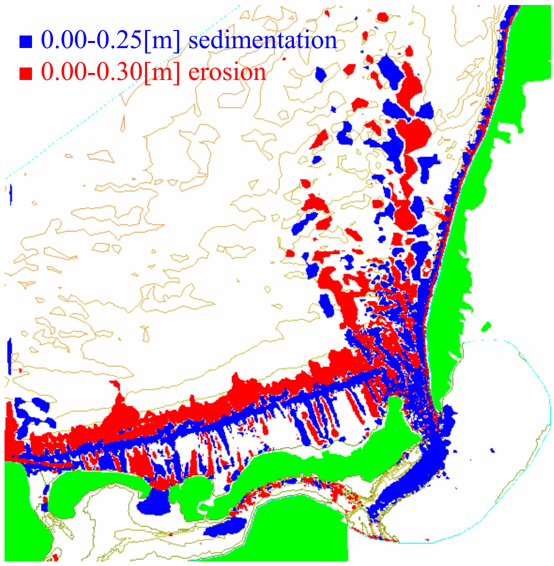


Figure 10 Computed sedimentation and erosion along a characteristic beach ridge

Conclusion

The recently developed holistic approach for modeling hydrodynamic and morphodynamic processes

- is free of data passing between conventional numerical process models,
- uses one single computational grid for all processes,
- is validated for currents and waves in the micro-tidal region of the Southern Baltic Sea,
- shows good agreement with observations regarding the formation of a beach ridge.

This model can be used as a predictive tool for morphodynamic studies in the coastal zone.

References

Brooks, A.N. and Hughes, T.J.R.(1982). Streamline upwinding / Petrov-Galerkin formulation for convection dominated flows with particular emphasis on the incompressible Navier-Stokes equations. *Computer Methods in Applied Mechanics and Engineering*, 32:199-259.

Kolmogorov, A.N. and S.V. Fomin, S.V. (1975) Reelle Funktionen und Funktionalanalysis, Deutscher Verlag der Wissenschaften

Lehfeldt, R., Milbradt, P., Zyserman, J.A., Barthel, V. (2002), Evaluation of simulation models used for morphodynamic studies, *ICCE2002, in press*

Milbradt, P. (1999), Directional wave propagation and Induced Currents in Coastal Regions, *Proc COPEDEC'99*, Capetown, pp192-201. CSIR, South Africa.

Milbradt, P. (1995) Zur mathematischen Modellierung großräumiger Wellen und Strömungsvorgänge. PhD-Thesis. University Hannover.