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B-SPLINE VOLUMES FOR TIME DEPENDENT BATHYMETRY MODELLING

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***Abstract**: In order to analyze the morphodynamic behaviour of coastal zones the interpolation of topographic measurement points over time is indispensable. Therefore, the concept to approximate bathymetries with parametric b-spline surfaces in three-dimensional space is extended to parametric b-spline volumes in four-dimensional space by introducing an additional parameter regarding time. B-spline volumes and even more general free form volumes are rarely used in science and practical applications*

At first, the mathematical basics are outlined in this contribution. The conventional bathymetry approximation with b-spline surfaces is illustrated for a small part of the Medem Cannel at the estuary of the River Elbe. This is performed for a single data set gained in the year 1985. The concept of time dependant bathymetry modelling is demonstrated for the same investigation area with additional data sets of the years from 1983 to 1988. The modelling concept based on b-spline volumes, which take time aspects into account, enables the user to trace any point of the bathymetry along the timescale.

***Keywords**: Bathymetry modelling; b-spline volumes; time dependent modelling.*

INTRODUCTION

Surface approximation based on free form surfaces is well established in engineering, especially in mechanical engineering. In general, the theoretical background of free form surfaces are single segment surfaces, such as Bernstein based Bézier surfaces, or multi segment surfaces, such as b-spline surfaces.

The application of free form surface techniques in the area of bathymetry approximation is shown for instance in Rath et al. (2004) and Berkhahn and Mai (2004). The smoothing property and the regular, two-dimensional parameter space are used to generate hybrid meshes for hydrodynamic simulations. These hybrid bathymetry meshes consist of regular as well as irregular quadrilateral or triangular elements.

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In the case of modelling bathymetries with respect to time, the concept of a two dimensional parameter space has to be augmented by the third parameter time. This leads to a time dependent representation of the bathymetry. In order to facilitate the understanding of the 4-dimesional case, a short overview of the mathematical basics of b-spline surface is given in the next section.

MATHEMATICAL BASICS OF B-SPLINE SURFACES

Surface approximation based on free form surfaces is well established in engineering, especially in mechanical engineering. Bézier curves and surfaces fulfil the requirement of a smooth approximation of geometry. In general, Bézier surfaces and the corresponding Bernstein polynomials are applicable to approximate bathymetries if the smoothing property is requested. However, for bathymetry approximations based on a large number of scattered data Bézier surfaces show the disadvantageous property of global modelling. This means, a single control point of a Bézier surface influences the characteristics of the entire surface. In order to avoid this disadvantageous property, the concept of Bézier surfaces is generalized to the concept of segmented surfaces, which leads to the surface representation with b-spline technique explained in detail by Farin (2002). A point on a b-spline surface is defined by

$$\mathbf{b}(u,v) = \begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix} = \sum_{i=0}^{N} \sum_{j=0}^{M} \mathbf{d}_{ij} N_i^K(u) N_j^L(v) \quad \text{with} \quad \mathbf{b}, \mathbf{d}_{ij} \in E^3 . \tag{1}$$

In equation (1) all expressions in bold face indicate a point in the three dimensional Euclidian space E^3. On the left hand side of this equation $\mathbf{b}(u,v)$ denotes a point on the b-spline surface in dependence of the two parameters u and v. The first expression $\mathbf{d}_{ij}$ in the double sum describes a regular grid of $N+1$ control points in u parameter direction and $M+1$ control points in v parameter direction. These control points are called de Boor points. The shape functions in u and v parameter directions are called b-spline functions $N_i^K(u)$ and $N_j^L(v)$, where the upper indices K and L indicate the degree of the b-spline functions.

To ensure the property of local modelling the influence of the control points with respect to the shape of the surface has to be restricted to a specified parameter range. Therefore the b-spline functions of degree 0 are defined as follows

$$N_i^0(u) = \begin{cases} 1 \text{ for } u \in [u_i, u_{i+1}[\\ \quad 0 \text{ else} \end{cases} \quad \text{for} \quad i = 0, \ldots, N+K \ . \tag{2}$$

In equation (2) u_i and u_{i+1} denote the lower and upper bounds of the ith parameter interval. All bounds of the parameter intervals are gathered in the knot vector $\mathbf{u}$

$$\mathbf{u} = [u_0, \ldots, u_{N+K+1}]^T \quad . \tag{3}$$

Any b-spline function $N_i^r(u)$ of degree r are given with the recursive formula (4), which are based on b-spline functions $N_i^{r-1}(u)$ and $N_{i+1}^{r-1}(u)$ of degree r-1. For the quadratic case the b-spline functions are shown in Figure 1.

$$N_i^r(u) = \frac{u - u_i}{u_{i+r} - u_i} N_i^{r-1}(u) + \frac{u_{i+r+1} - u}{u_{i+r+1} - u_{i+1}} N_{i+1}^{r-1}(u) \tag{4}$$

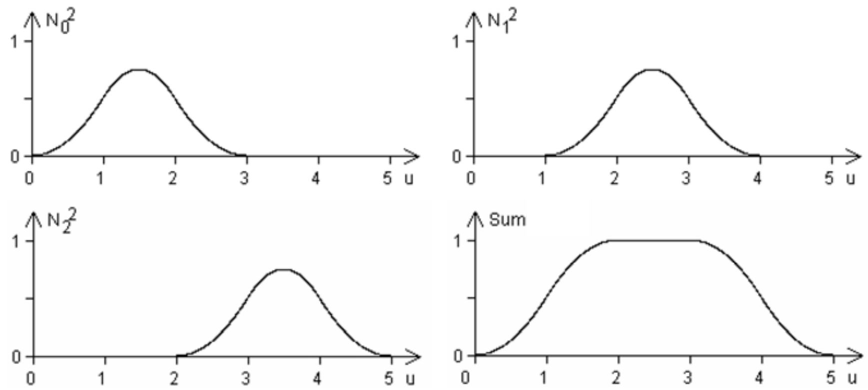

Figure 1. Quadratic b-spline functions $N_i^2(u)$ with equidistant knots and the sum of b-spline functions.

In general, the margins of a b-spline curve or surface are not equal to the margins of the de Boor polygon or grid. In order to guarantee this important property of end-point interpolation, multiple end-knots given by equation (5) are defined in the knot vectors. The corresponding quadratic b-spline functions are shown in Figure 2.

$$u_0 = ... = u_K \quad \text{and} \quad u_N = ... = u_{N+K+1} \quad . \tag{5}$$

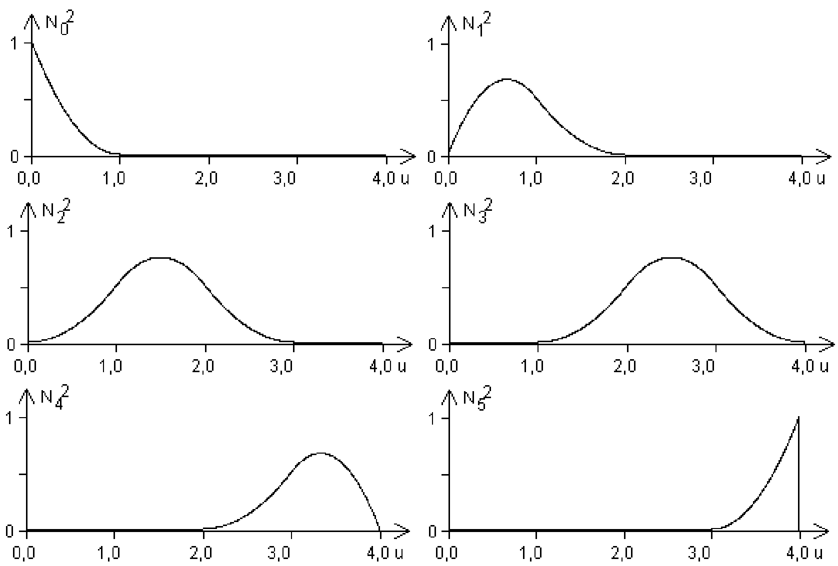

Figure 2. Quadratic b-spline functions $N_i^2(u)$ with multiple end-knots.

The b-spline functions $N_i^r(u)$ ensure the important property of local modelling indicated by equation (6)

$$N_i^r(u) = 0 \quad \text{for} \quad u \in R \setminus \left]u_i, u_{i+r+1}\right[\quad . \tag{6}$$

Equation (4) has to represent an affine combination in order to admit the addition of points. Thus, the factors of all control points for every parameter combination *u* and *v* have to summarize to 1. The sum of b-spline functions in the quadratic case is shown in the last picture in Figure 1. This requirement is only fulfilled within the interval

$$\sum_{i=0}^{N} N_i^K(u) = 1 \quad \text{for} \quad u \in \left[u_K, u_{N+1}\right] \quad , \tag{7}$$

which leads to the restriction for the parameters *u* and *v* in definition (1)

$$\mathbf{b}(u,v) = \sum_{i=0}^{N}\sum_{j=0}^{M} \mathbf{d}_{ij} N_i^K(u) N_j^L(v) \quad \text{with} \quad \begin{matrix} \mathbf{b}, \mathbf{d}_{ij} \in E^3 \\ u \in \left[u_K, u_{N+1}\right] \\ v \in \left[v_L, v_{M+1}\right] \end{matrix} \quad . \tag{8}$$

BATHYMETRY APPROXIMATION BASED ON B-SPLINE SURFACES

The technique of bathymetry approximation is illustrated for area under investigation located at the estuary of the River Elbe. The so called Medem Channel is characterized by huge morphodynamic changes with respect to time. A small estuary area shown in figure 3 consists of bathymetry points measured in the year 1985.

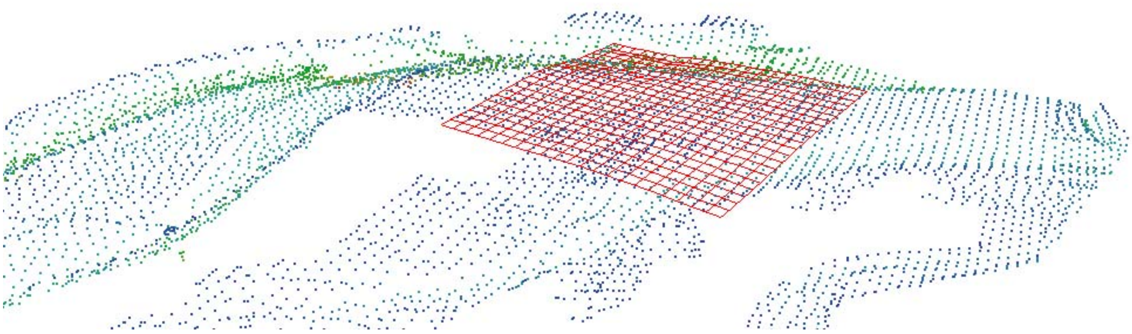

Figure 3. Measurement data (1985) of the Medem Channel at the estuary of the River Elbe and a control point grid of an approximating b-spline surface.

In addition, figure 3 shows a control point grid $\mathbf{d}_{ij}$ illustrated with red lines. For clearness reasons only a very small part of the entire area under investigation is chosen to be approximated by a b-spline surface, which is shown in figure 4. The geodetic height at any bathymetry location can be determined by the z component of equation (8).

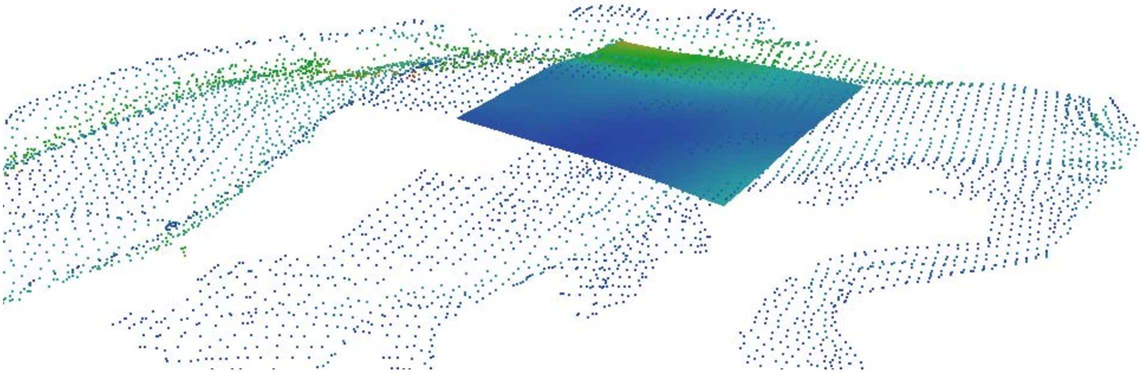

Figure 4. B-spline surface approximating the bathymetry.

TIME DEPENDENT BATHYMETRY MODELLING

If the morphodynamic changes with respect to time of a bathymetry are considered, it is evident to enhance equation (1) respective (8) from a surface representation to volume representation given by equation (9):

$$\mathbf{b}(u,v,t)=\begin{bmatrix} x(u,v,t) \\ y(u,v,t) \\ z(u,v,t) \\ T(u,v,t) \end{bmatrix} = \sum_{i=0}^{M}\sum_{j=0}^{N}\sum_{k=0}^{O}\mathbf{d}_{ijk}N_i(u)N_j(v)N_k(t) \quad \text{with} \quad \mathbf{b},\mathbf{d}_{ijk}\in R^4 \quad . \tag{9}$$

Volume points **b** and control points $\mathbf{d}_{ijk}$ are located in the four dimensional Euclidian space E^4 with x, y, z representing the location in space and T for the time. The set of parameters *u* and *v* is enhanced by the parameter *t* corresponding to the coordinate direction representing time.

The extension of free form surfaces to free form volumes enables the user to trace any point of the bathymetry along the timescale. As an illustrative example the measurement points of the Medem Channel of the year 1985 are extended by the results of measurement campaigns of the year 1983 to 1988 as shown in figure 5. For visualization purpose the points **b** and $\mathbf{d}_{ijk}$ in E^4 are projected into the reduced space E^3 by adding the z and T coordinates.

In addition to the measurement points, the different layers of the control point volume $\mathbf{d}_{ijk}$ are illustrated in figure 5 by red grid, where each displayed grid corresponds to a specific origin year. Figure 6 shows different illustration of the b-spline volume representing the time dependent bathymetry.

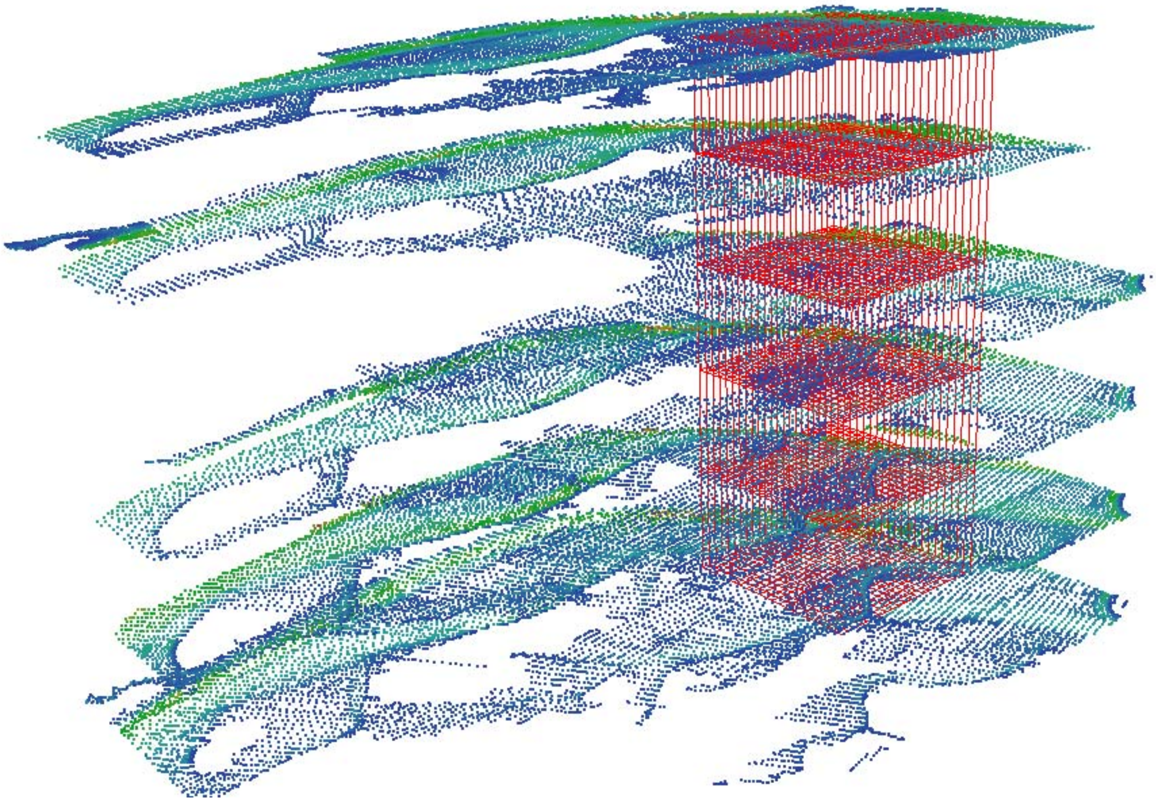

Figure 5. Measurement data (1983 - 1988) of the Medem Channel and a control point grid volume of an approximating b-spline volume.

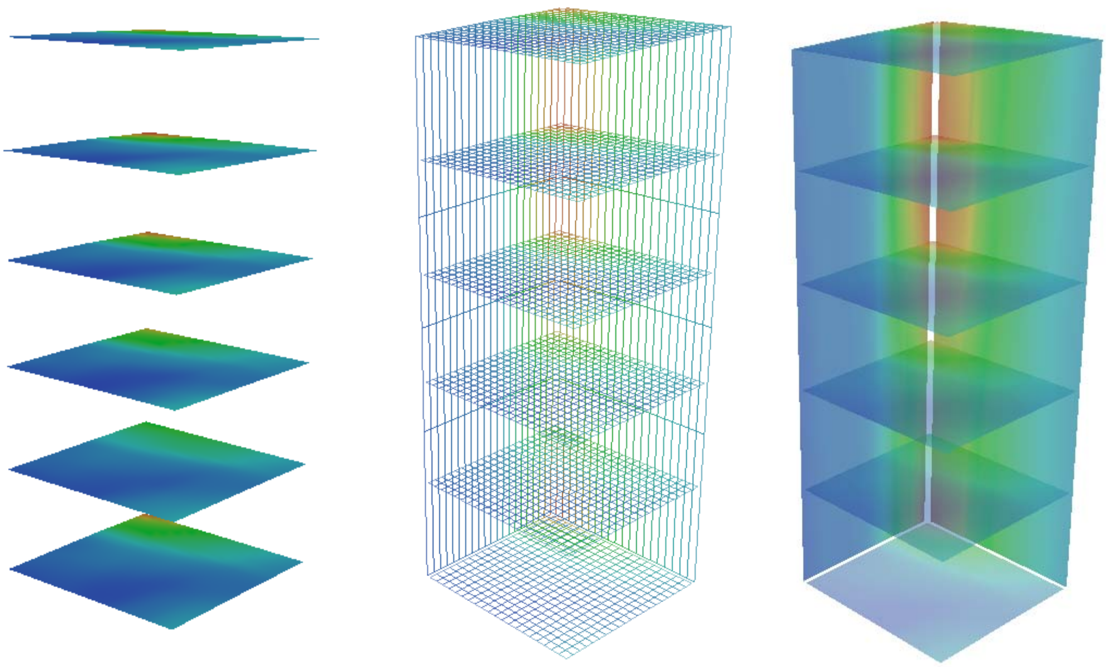

Figure 6. Different illustration of a b-spline volume approximating the measurement points (1983 - 1988) of the Medem Channel.

CONCLUSIONS

The concept of bathymetry modelling based on b-spline surface is enhanced to a time dependant bathymetry modelling by introducing b-spline volumes, which are rarely used in science and practical applications. The applicability of b-spline volume based bathymetry modelling is demonstrated for an investigation area with data set of the years from 1983 to 1988. This concept enables the user to trace any point of the bathymetry along the timescale.

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